Applications of Matched Filtering for Improved Interferometric Imaging and Weak Line Detection

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Collaborators: K. Öberg, S. Andrews, C. Walsh, I. Czekala, J. Huang

Improvements with new interferometers

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- Spatial resolution

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- Sensitivity

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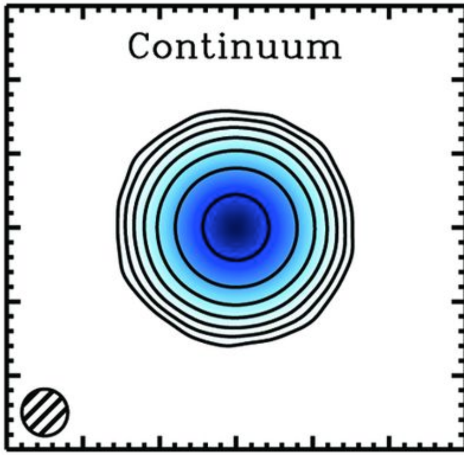
TW Hya



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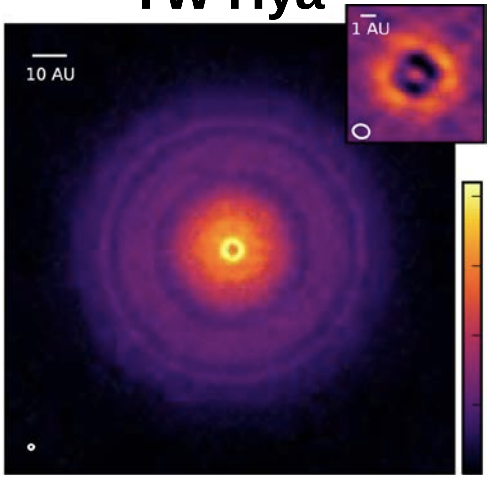

Andrews + (2016)

Improvements with new interferometers

- Spatial resolution
- Sensitivity
- Bandwidth

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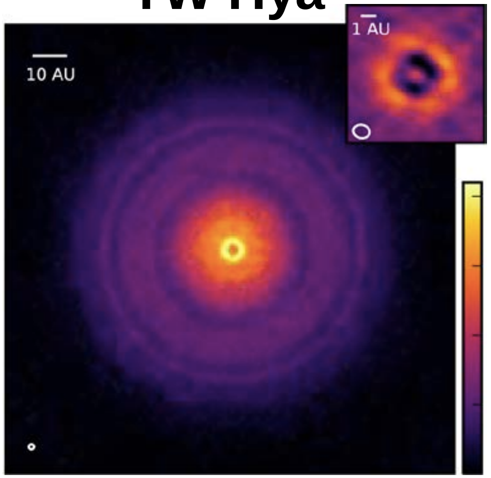

Andrews + (2016)

Improvements with new interferometers

- Spatial resolution
- Sensitivity
- Bandwidth
- Spectral resolution

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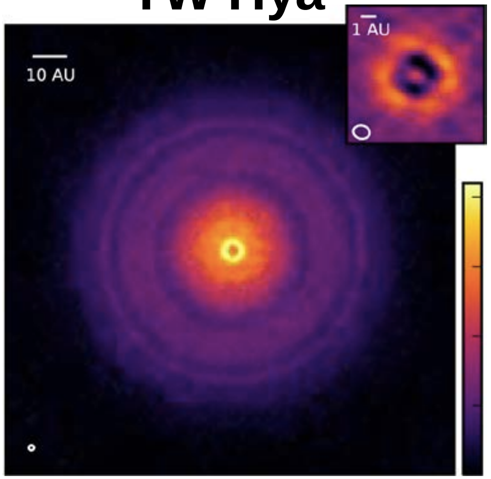

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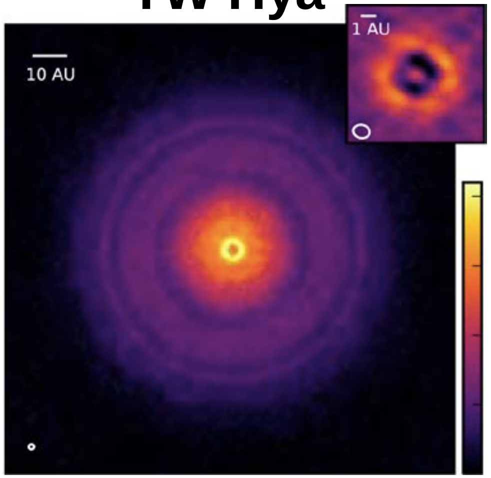

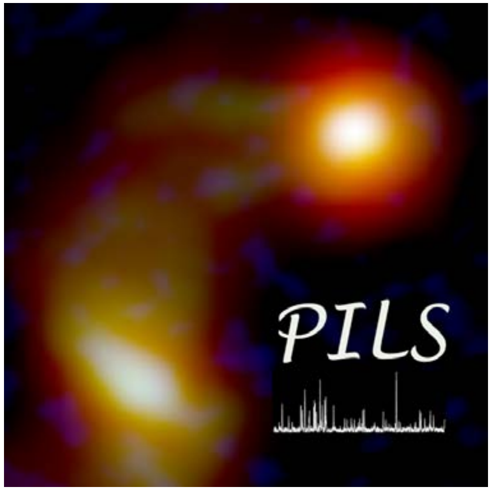

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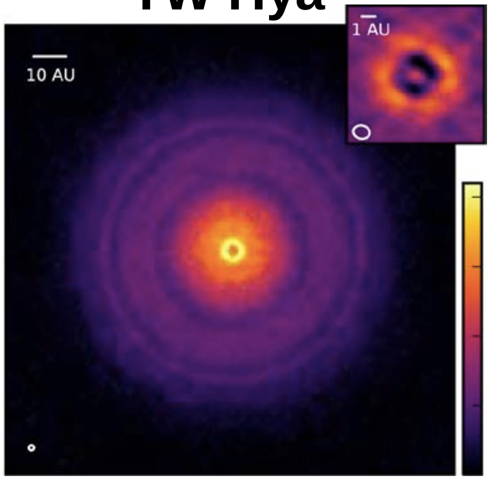
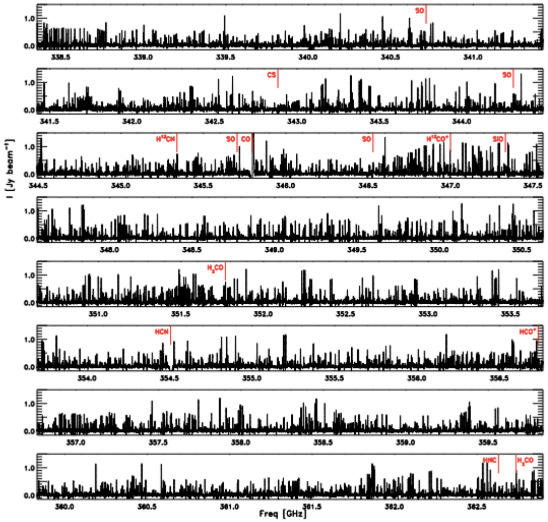


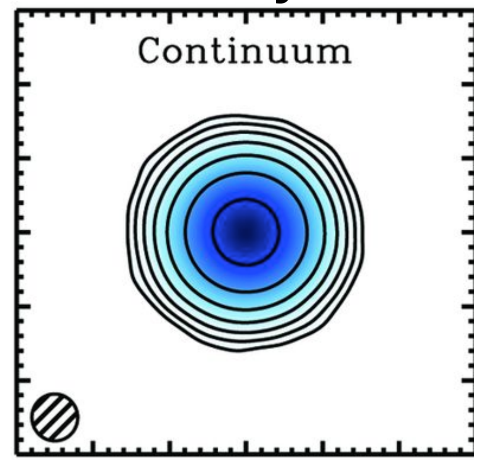


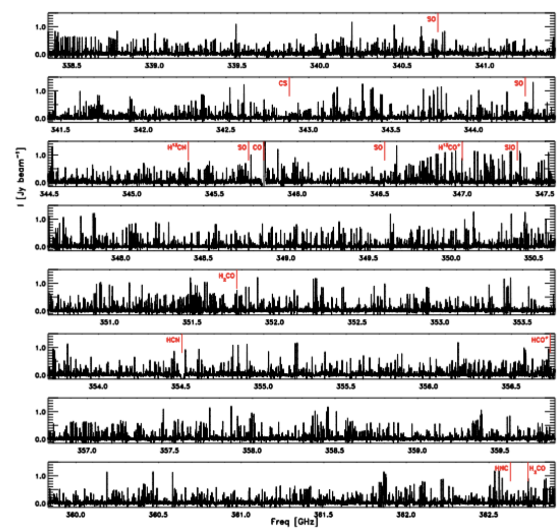
Jorgensen + (2016)

Point source assumptions

- Spatial resolution
- Sensitivity
- Bandwidth
- Spectral resolution

TW Hya





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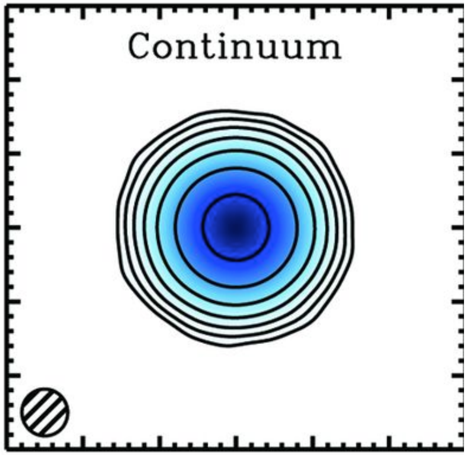
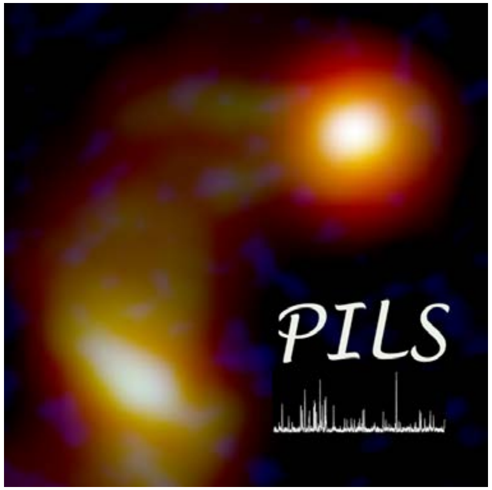
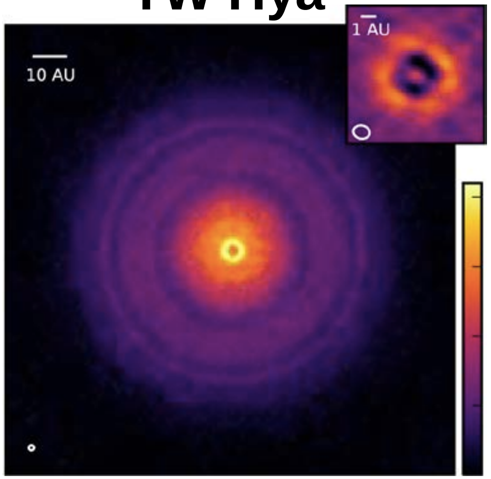


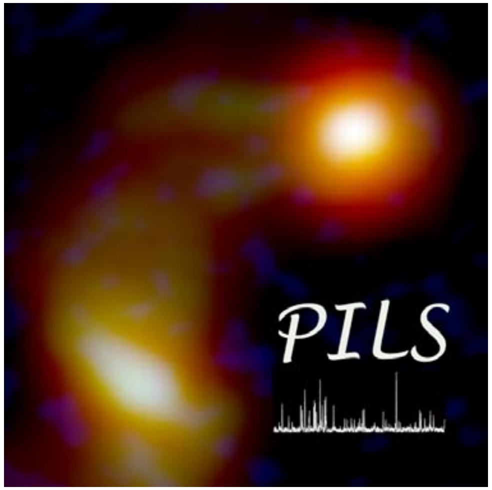


Opportunities for improved techniques

- Spatial resolution
- Sensitivity
- Bandwidth
- Spectral resolution

TW Hya



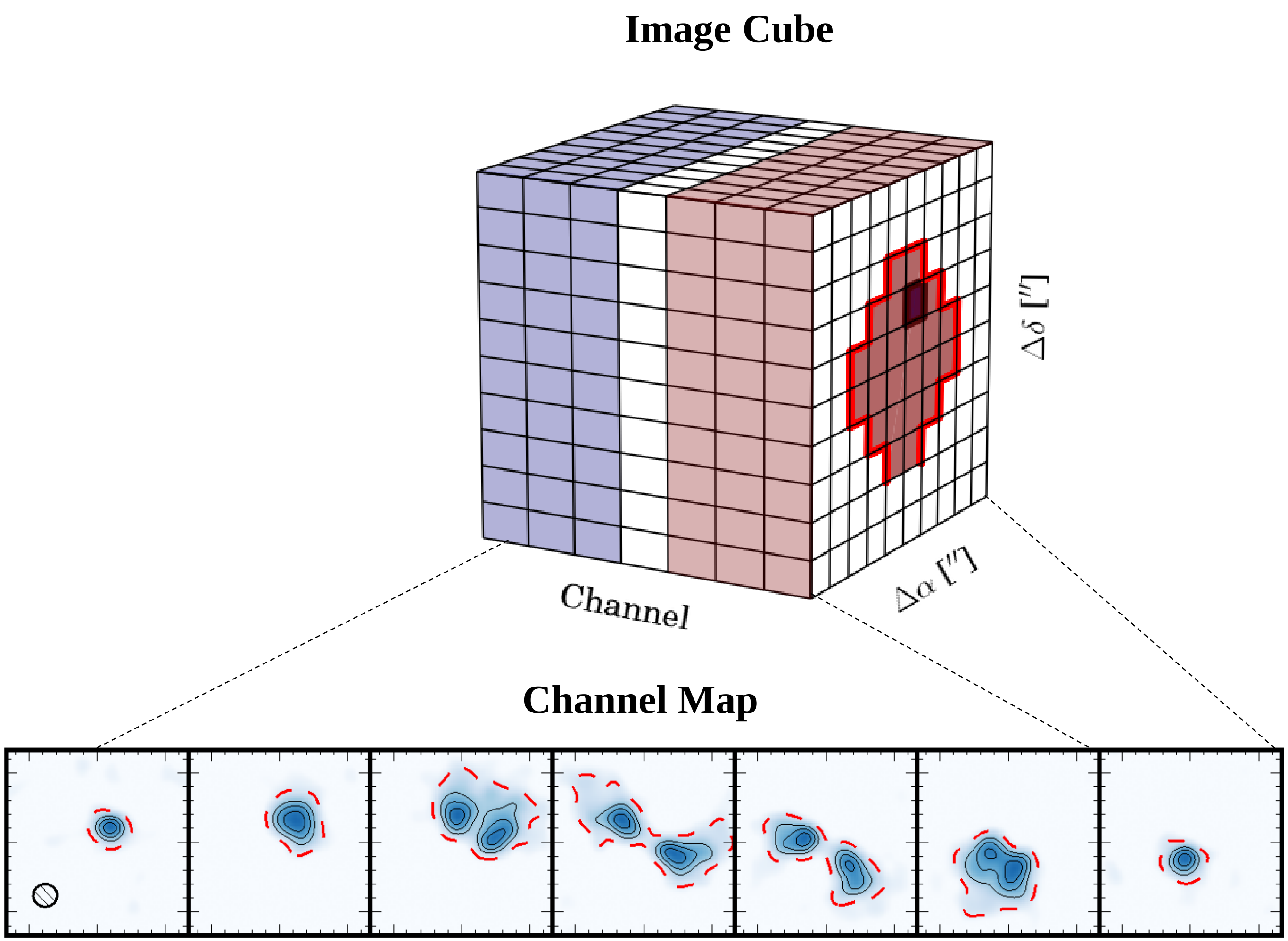


Outline

- Matched filtering for improved weak spectral line detection

- Generalized Briggs weighting for improved image fidelity

Analyzing imaged observations



[Loomis + 2018]

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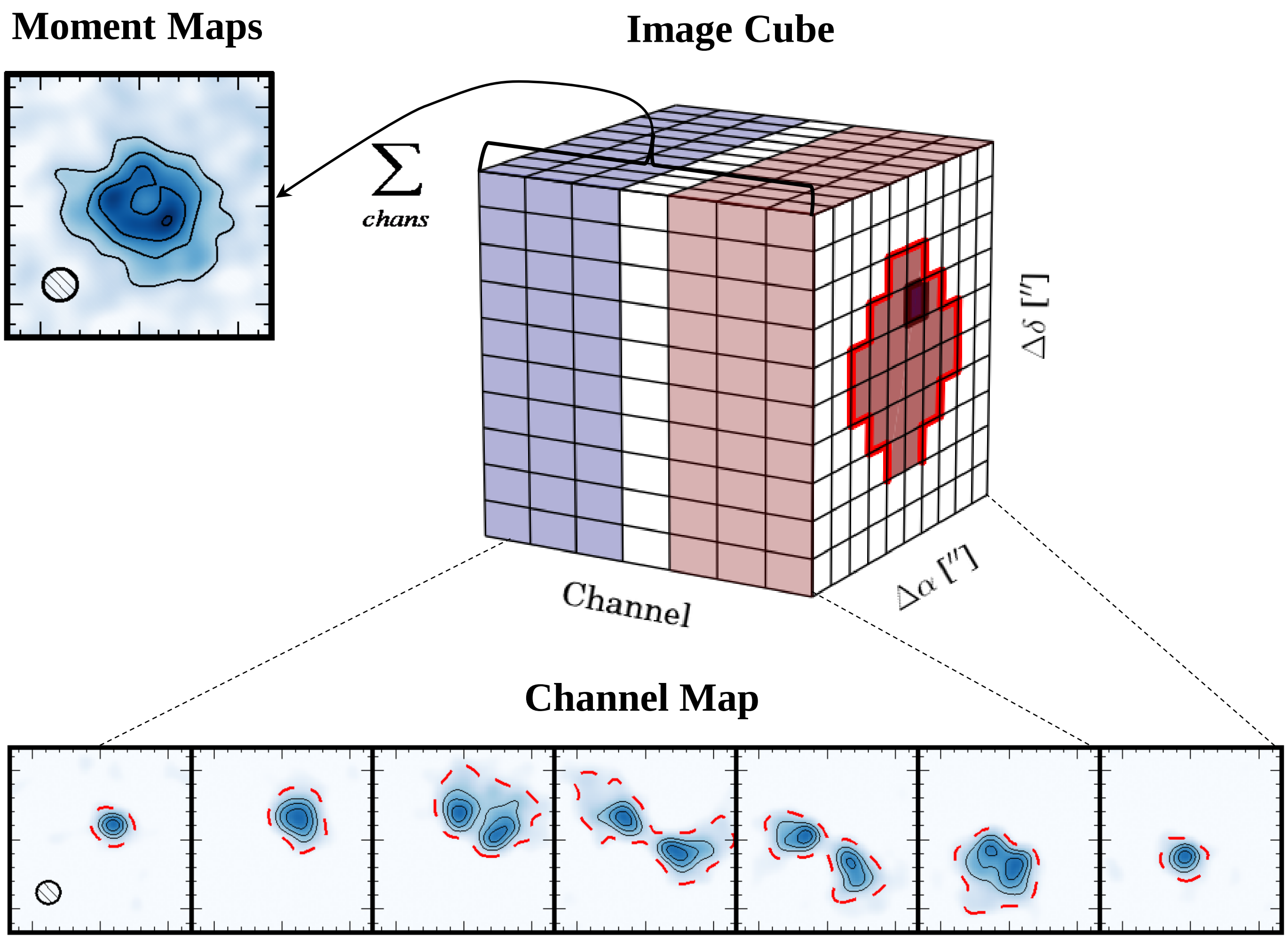

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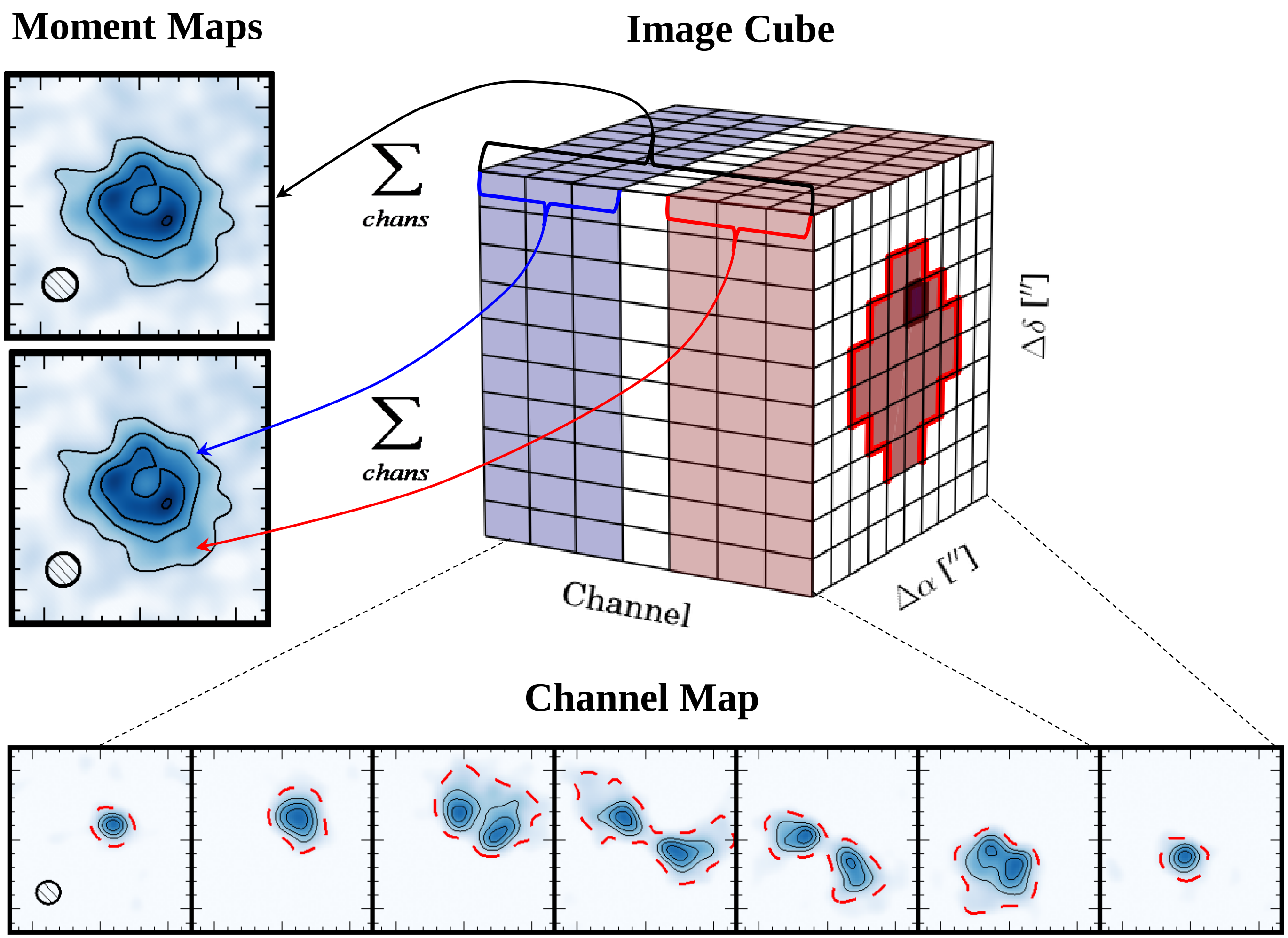

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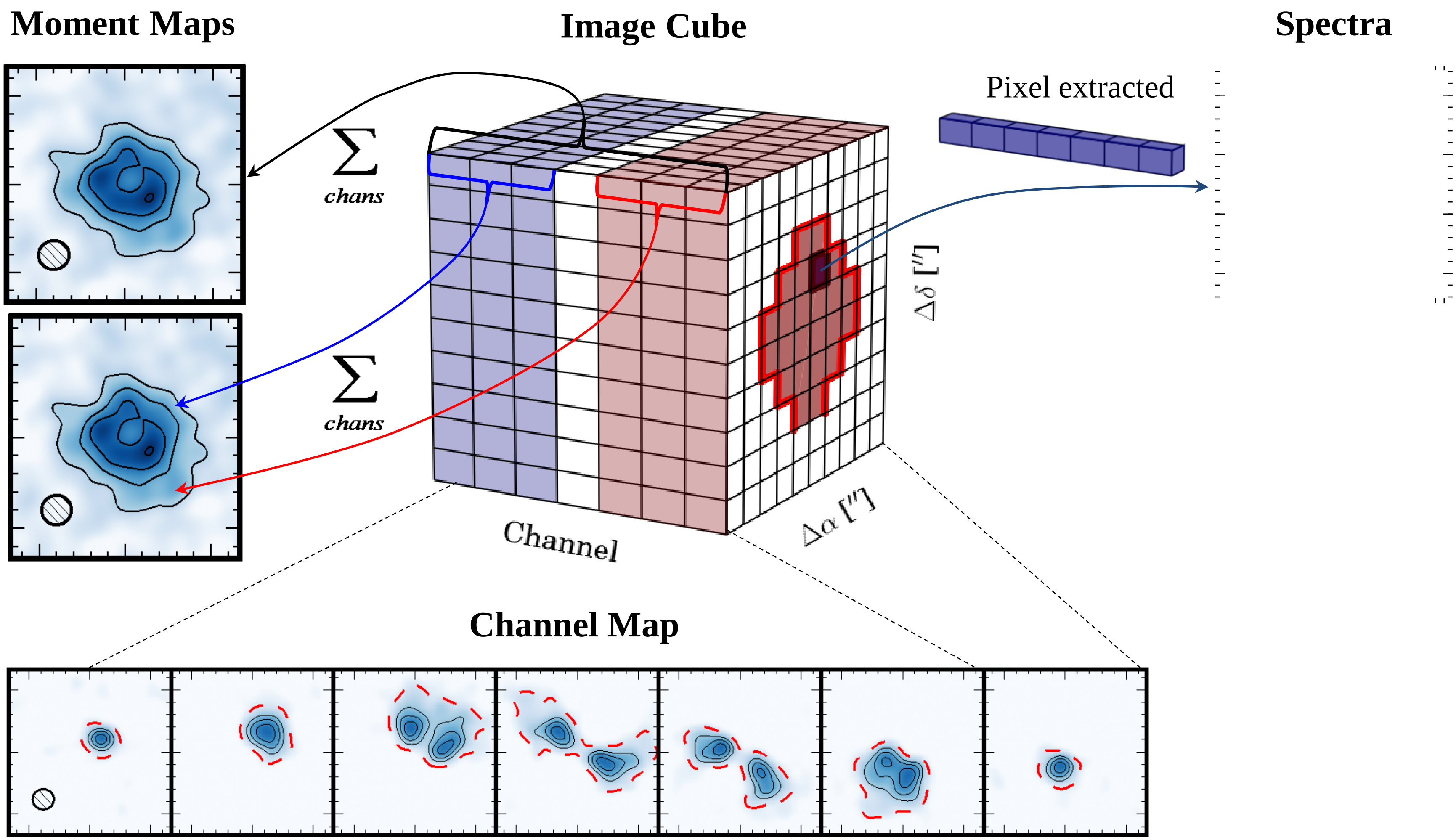

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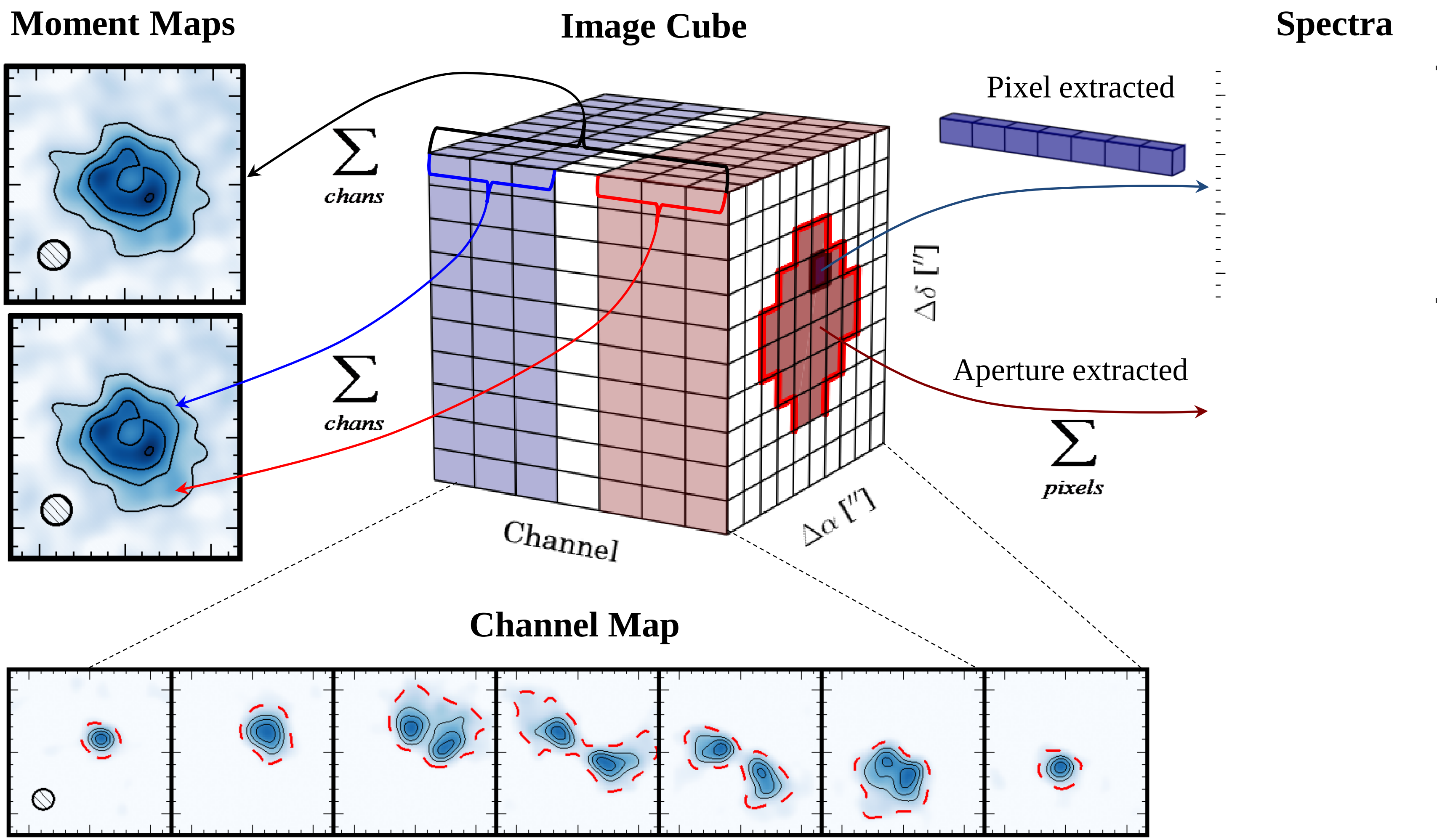

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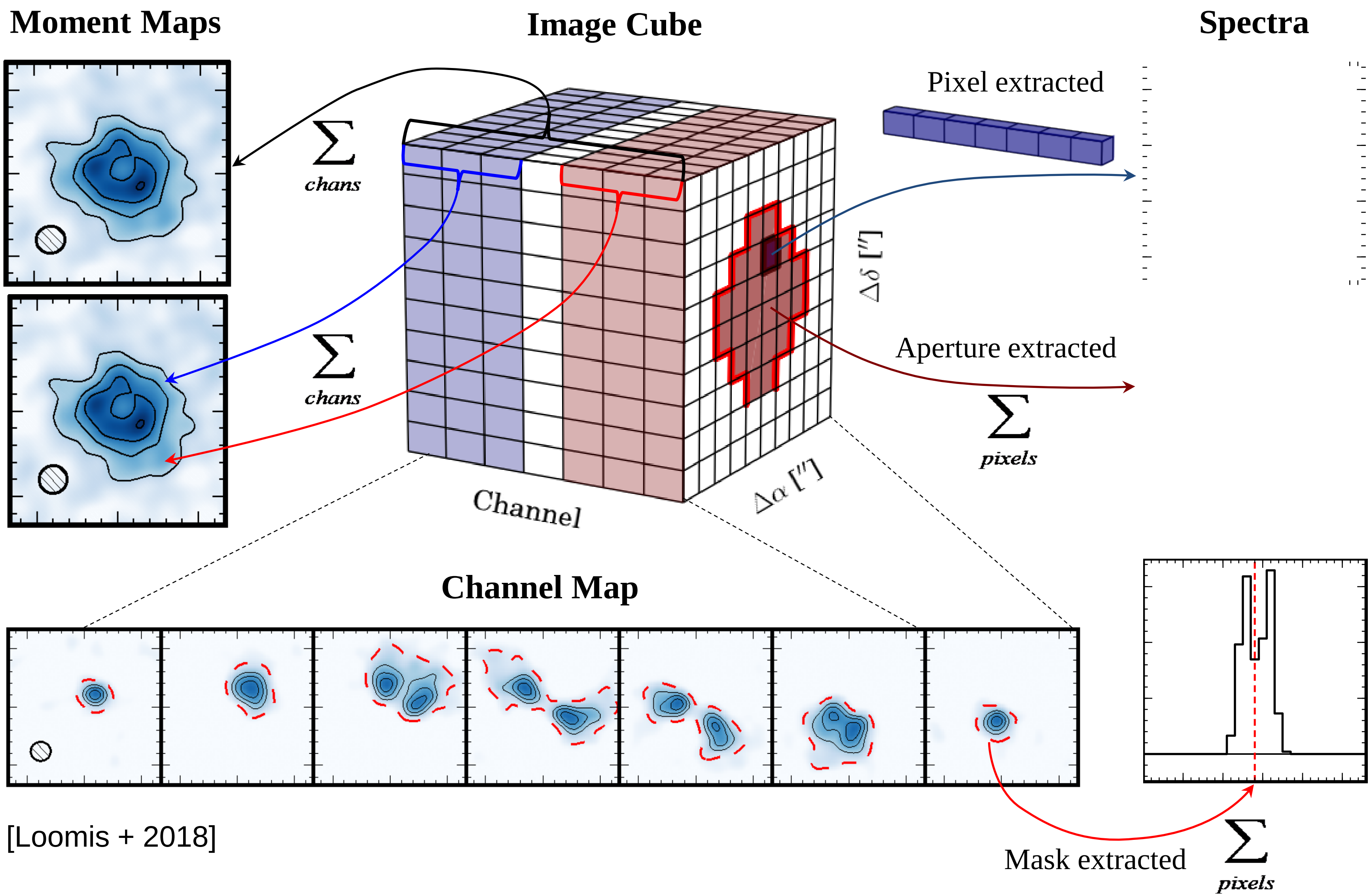
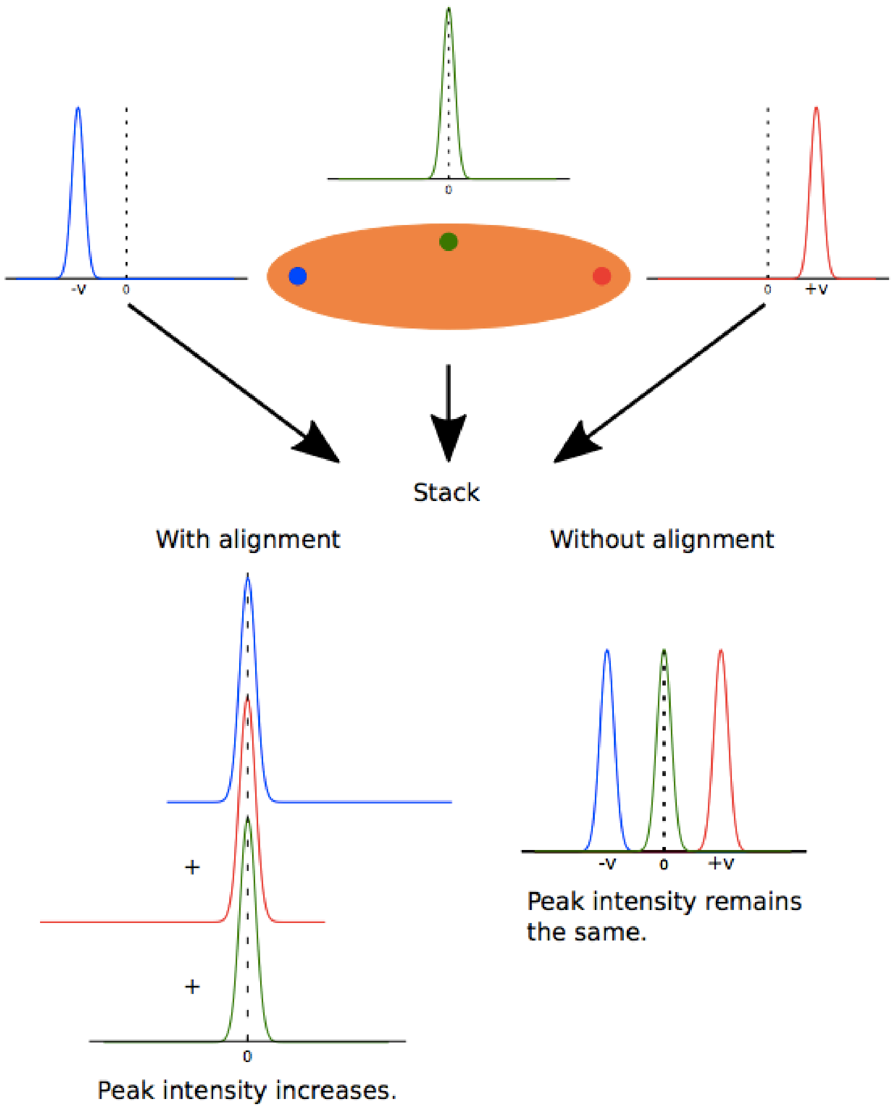
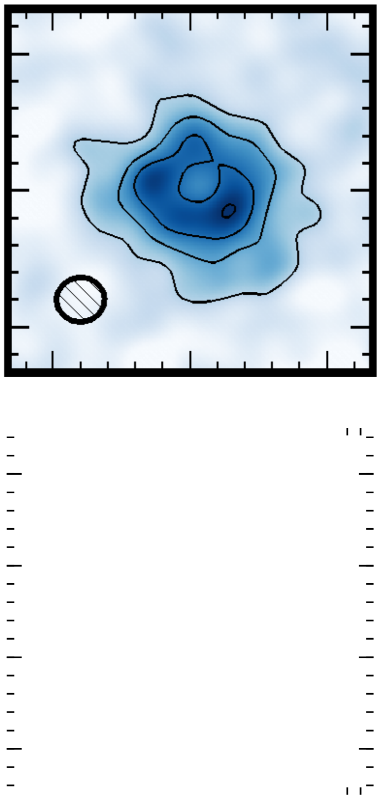


[Loomis + 2018]

Spectral stacking





Matra + 2015

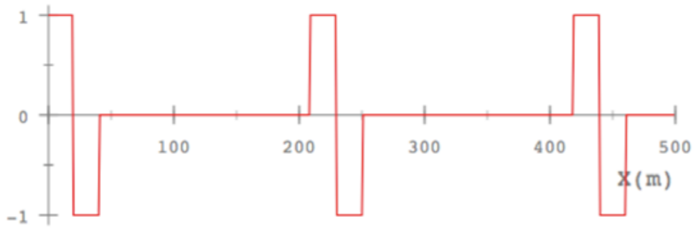
Marino + 2016

Yen + 2016

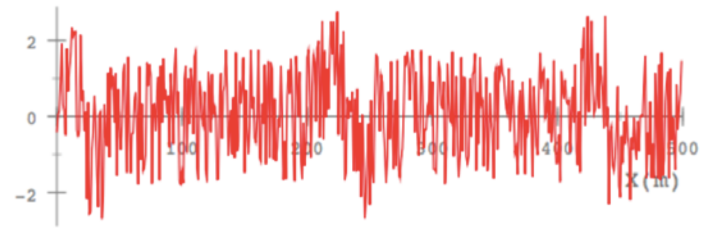
Matched filtering

Convolve known template signal with noisy data

Signal (used as filter/kernel)



Noisy signal



http://www.grauonline.de/alexwww/ardumower/filter/filter.html https://losc.ligo.org/tutorial_optimal/

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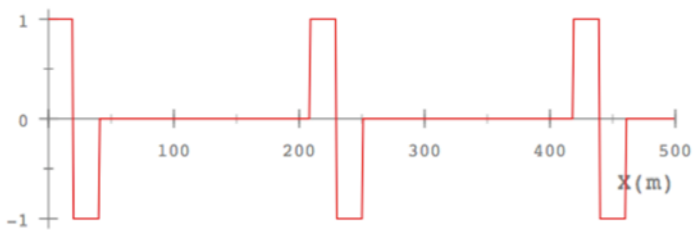

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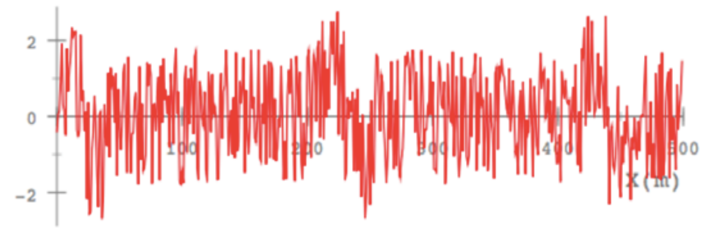

$$T_{i_0} = \sum_{i=i_0}^{i_0+N_s-1} x_i h_{i-i_0}; \qquad i_0 = 0, 1, ..., N - N_s$$

$$h = \frac{1}{s^H R_v^{-1} s} R_v^{-1} s$$

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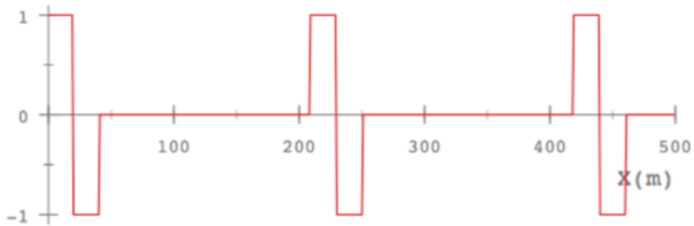

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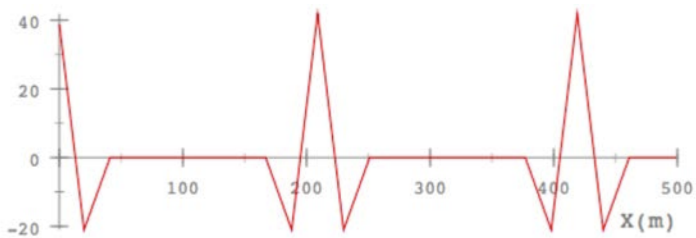
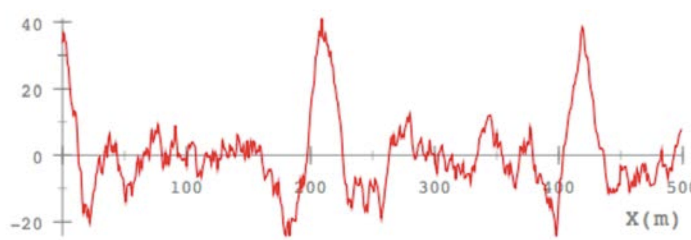
Filter response has highest recoverable SNR

Autocorrelation of signal



Filter response



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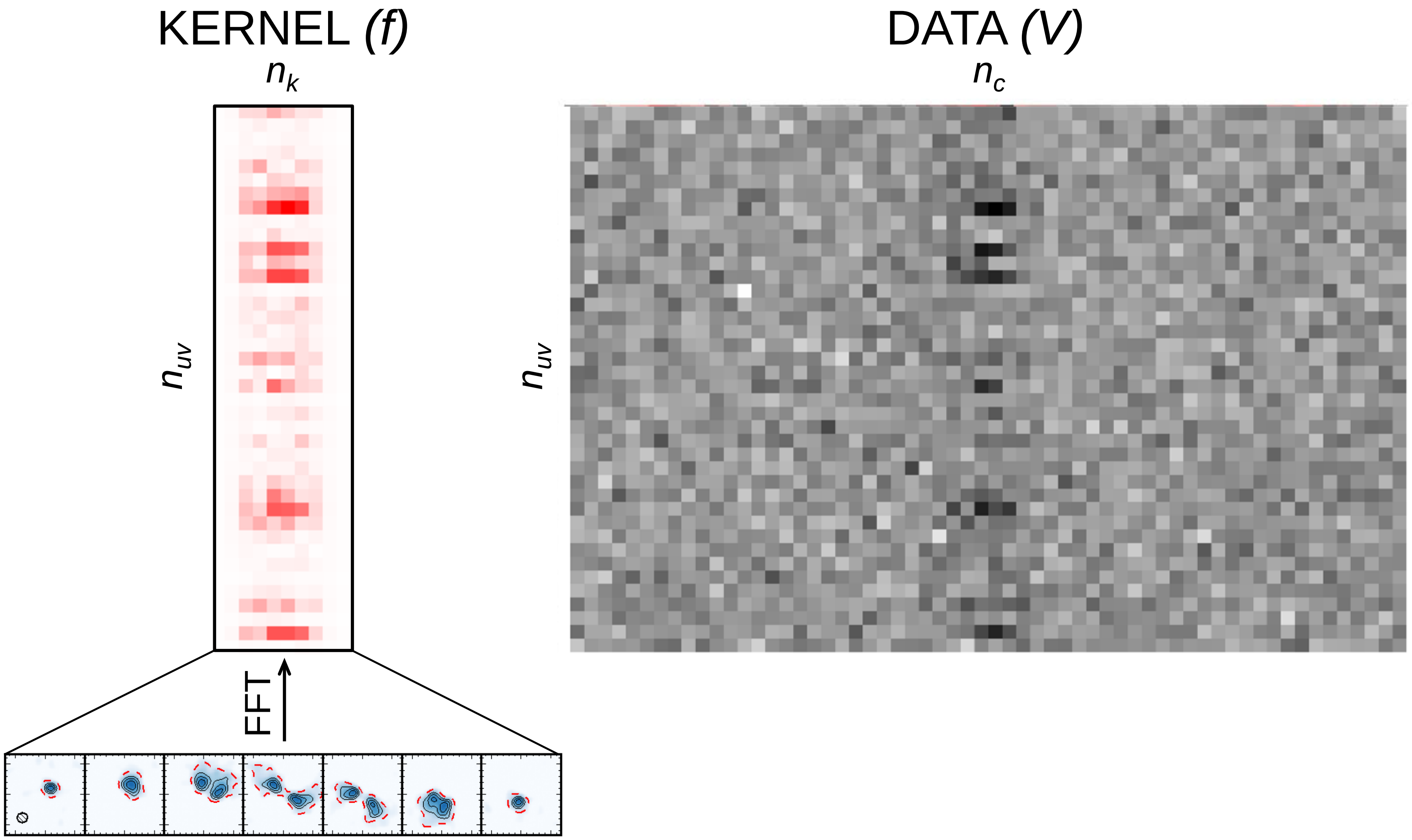
Matched filtering interferometric data



[Loomis + 2018]

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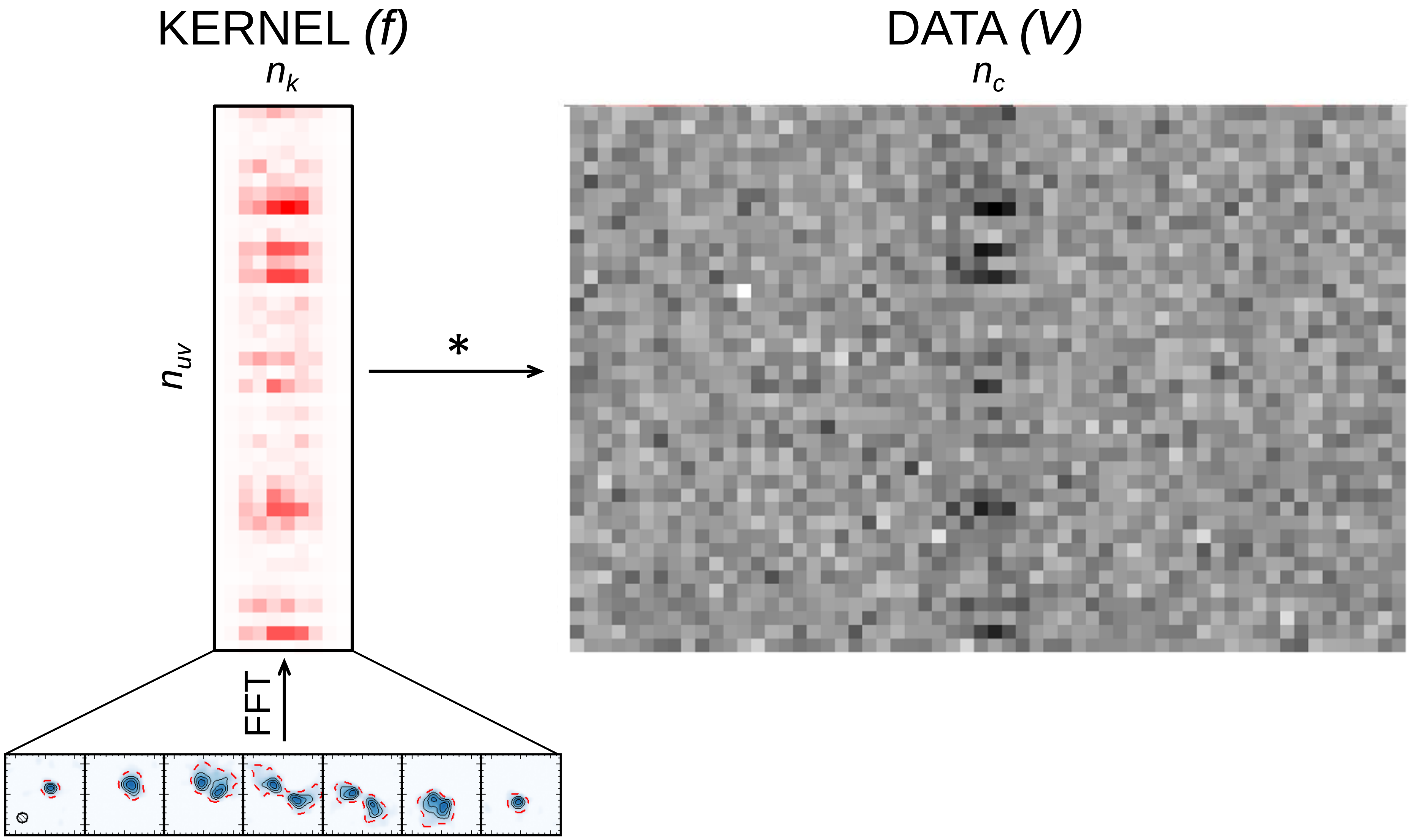

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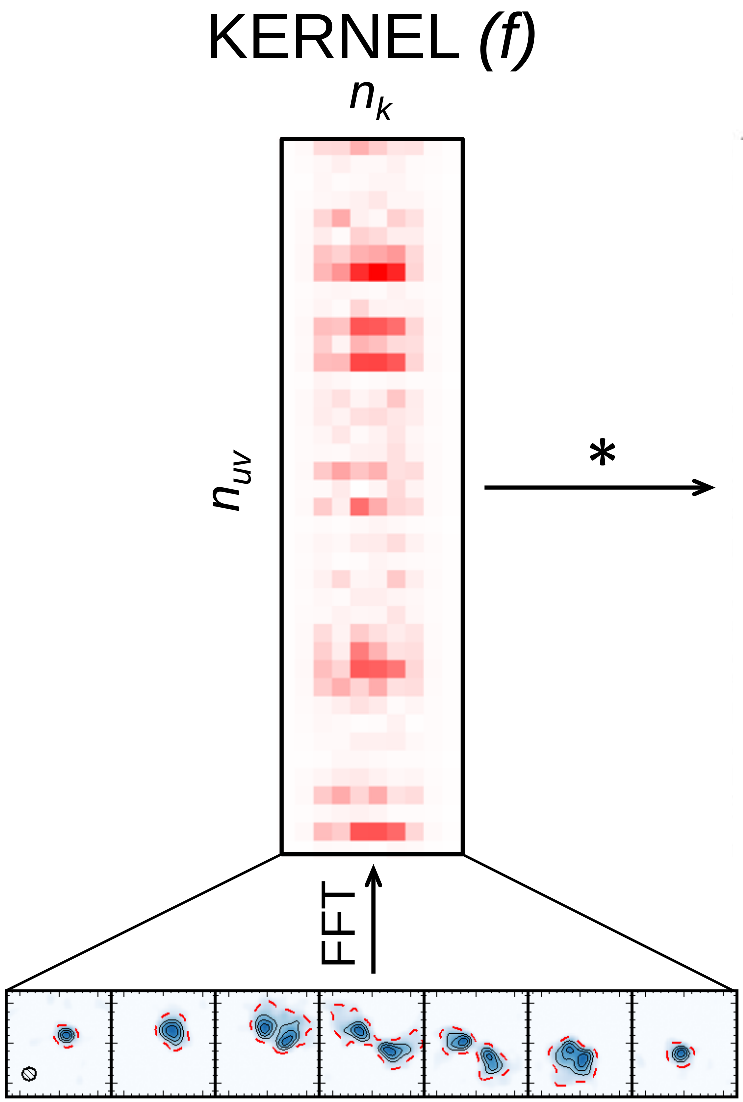
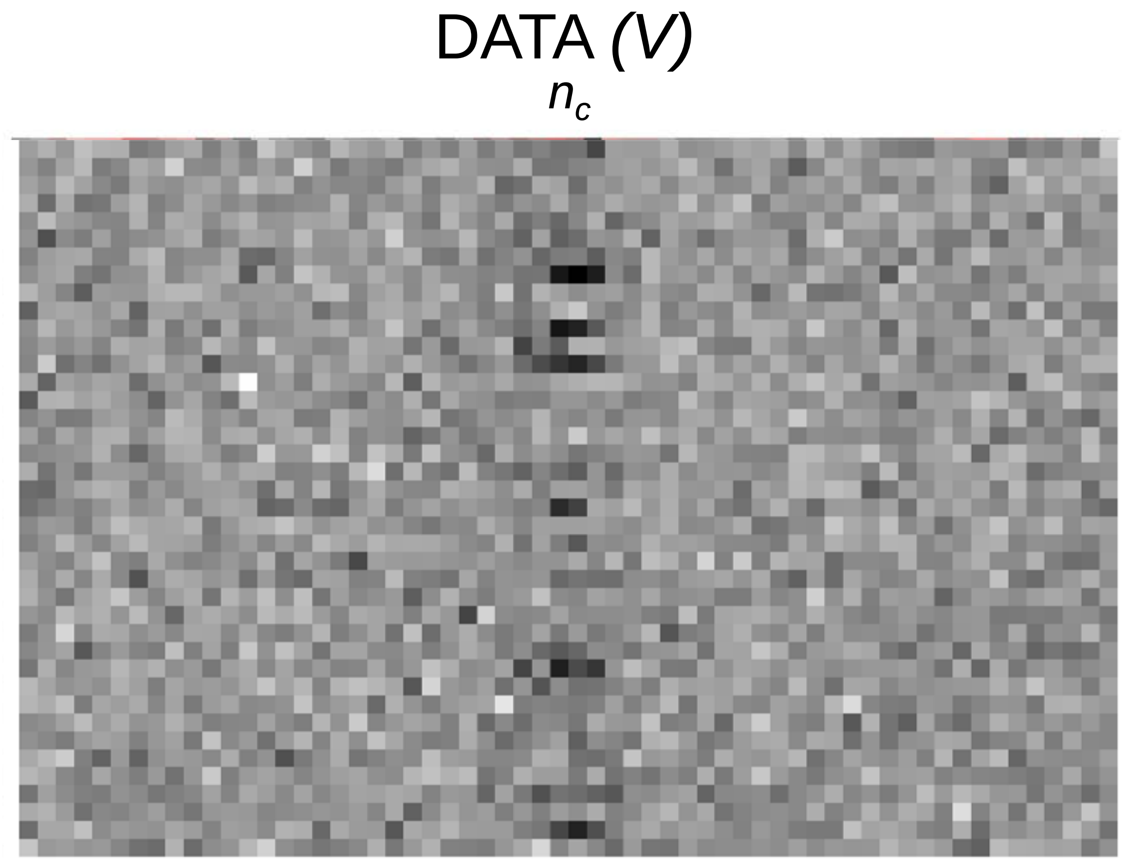


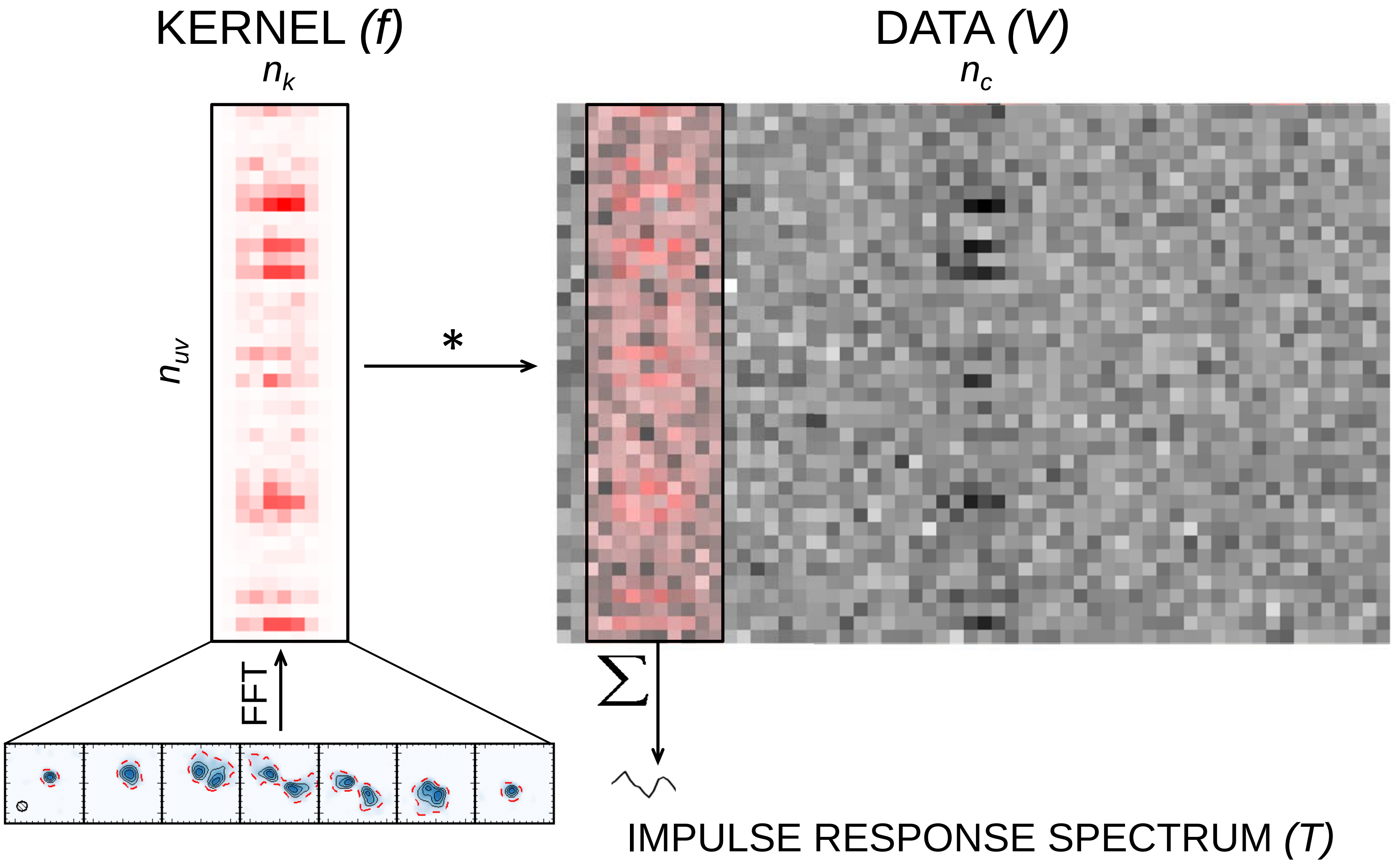


$$T_{i_0} = \mathrm{Re}\left[\sqrt{2}\sum_{i=i_0}^{i_0+n_k-1}\sum_{j=0}^{n_{uv}} V_{i,j}h_{i-i_0,j}\right]; \quad i_0 \in [0, n_c - n_k],$$

IMPULSE RESPONSE SPECTRUM *(T)*

[Loomis + 2018]

Matched filtering interferometric data



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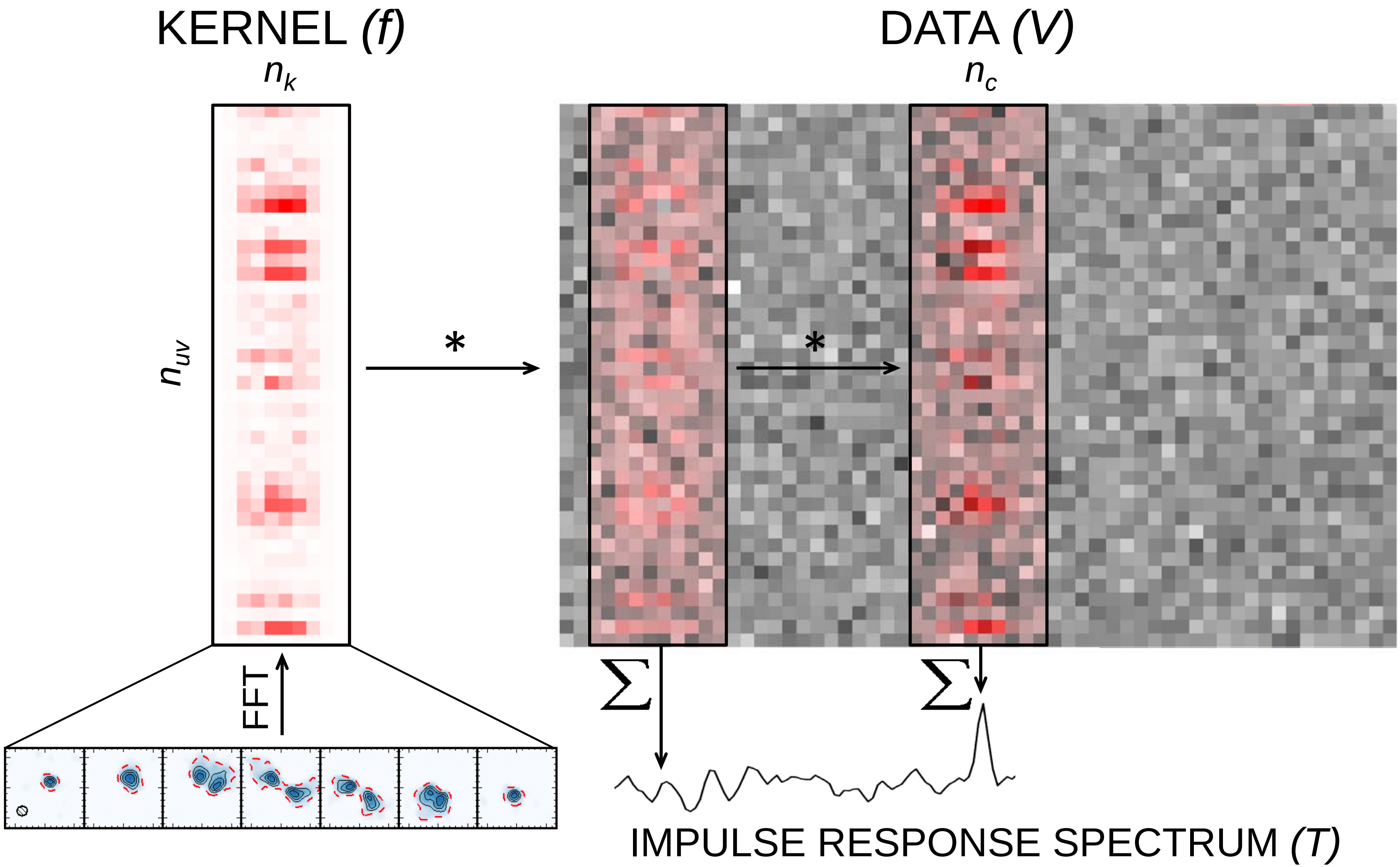

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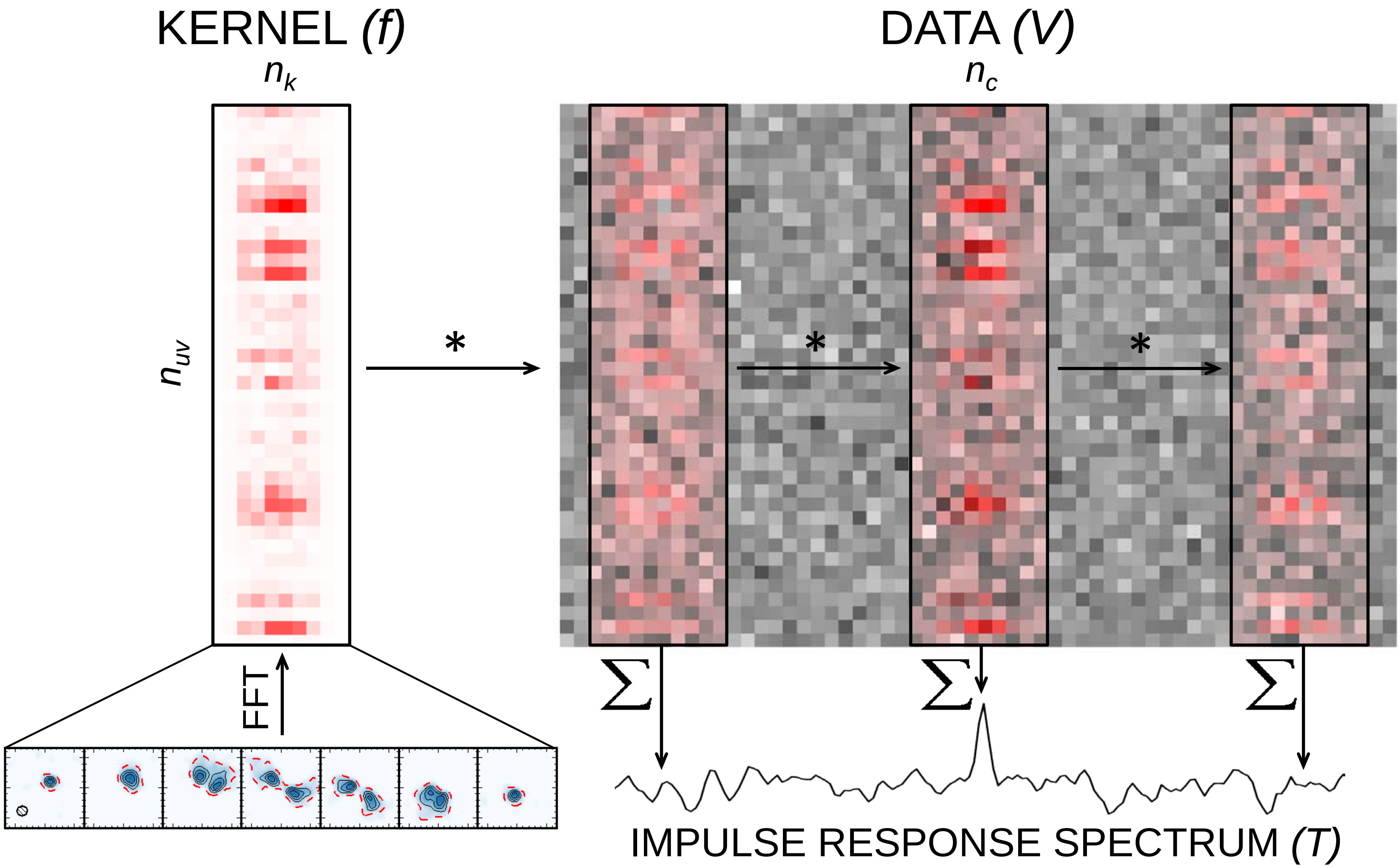
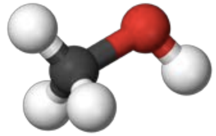
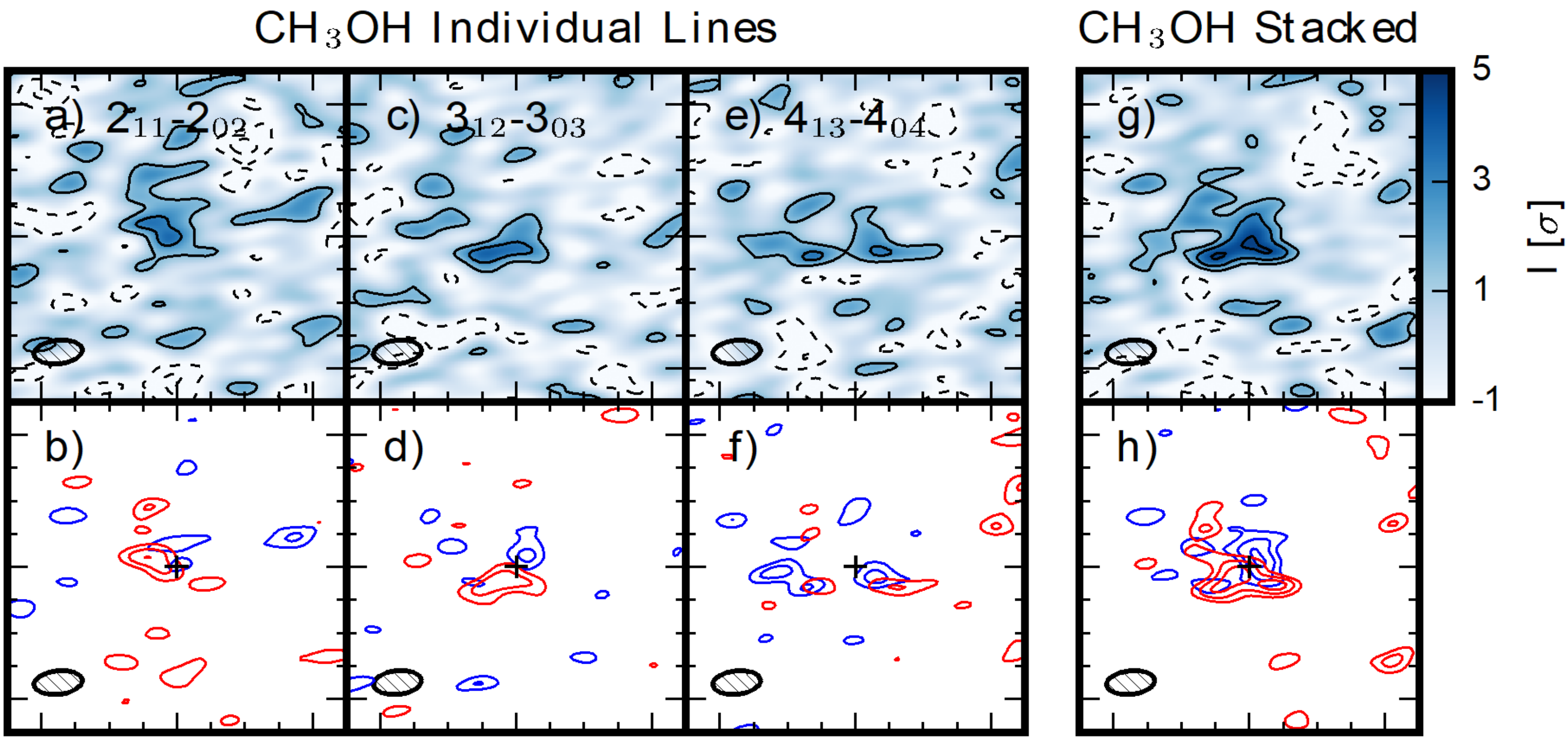


[Loomis + 2018]

Test case #1 – CH_3OH in TW Hya





[Walsh, Loomis, Öberg + 2016]

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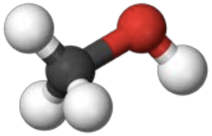

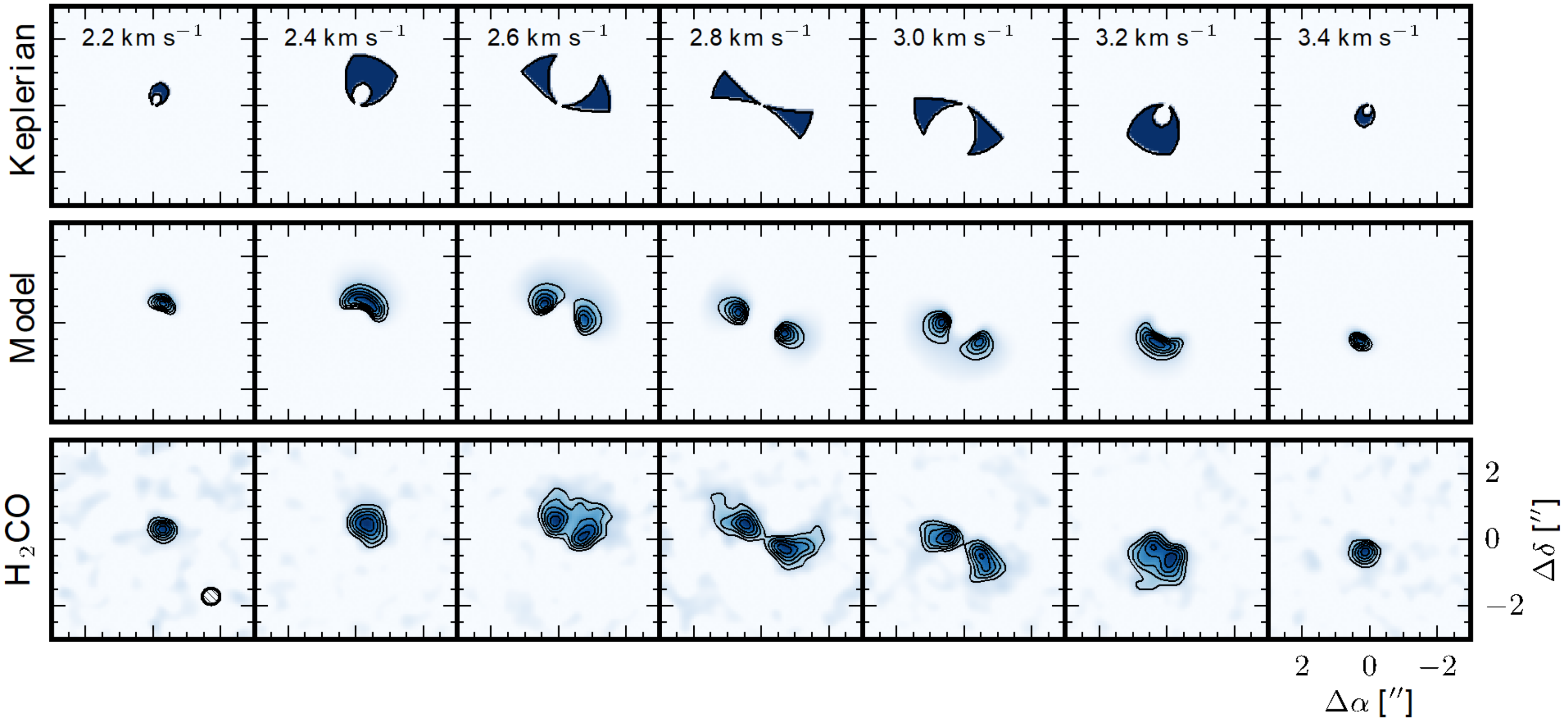

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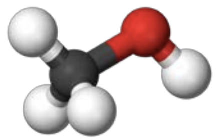
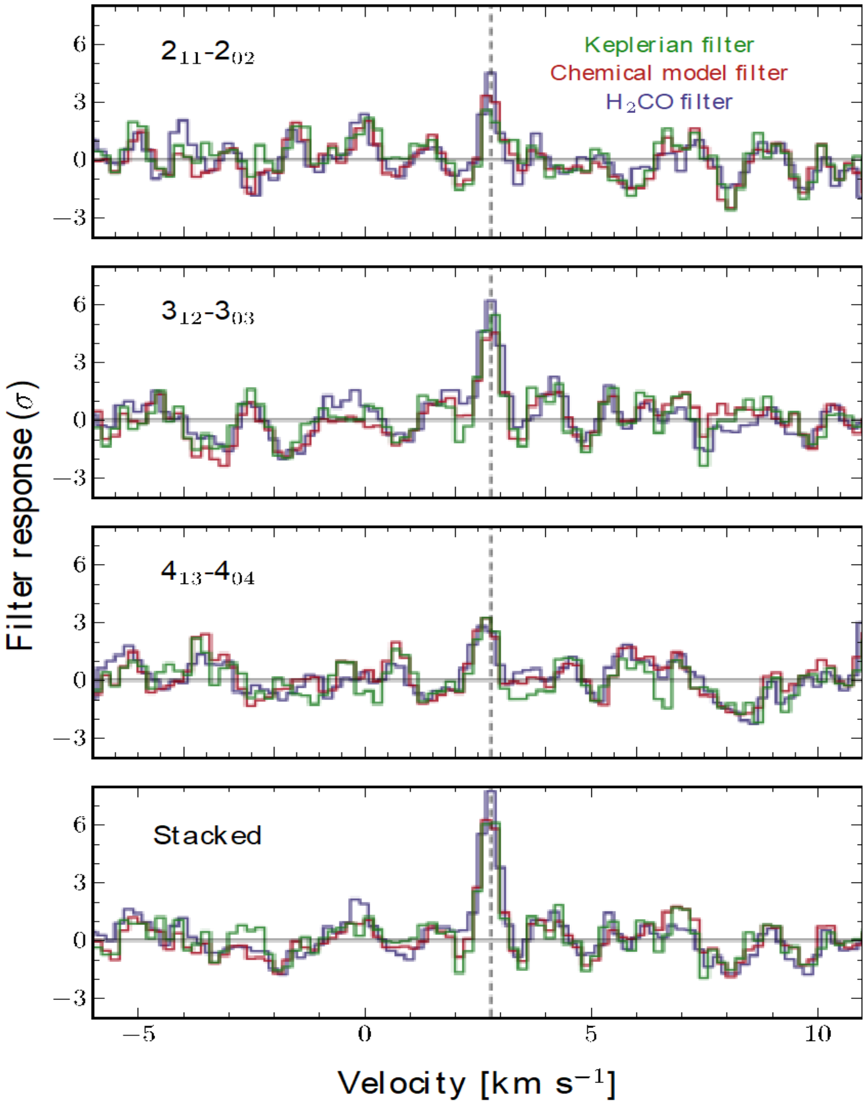
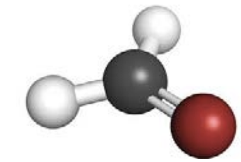


- Individual lines are now detected
- SNR significantly enhanced – 7.8σ after matched filtering vs 5.1σ before (53% improvement)
- 53% SNR boost is equivalent to a factor of 2.3 in observing time

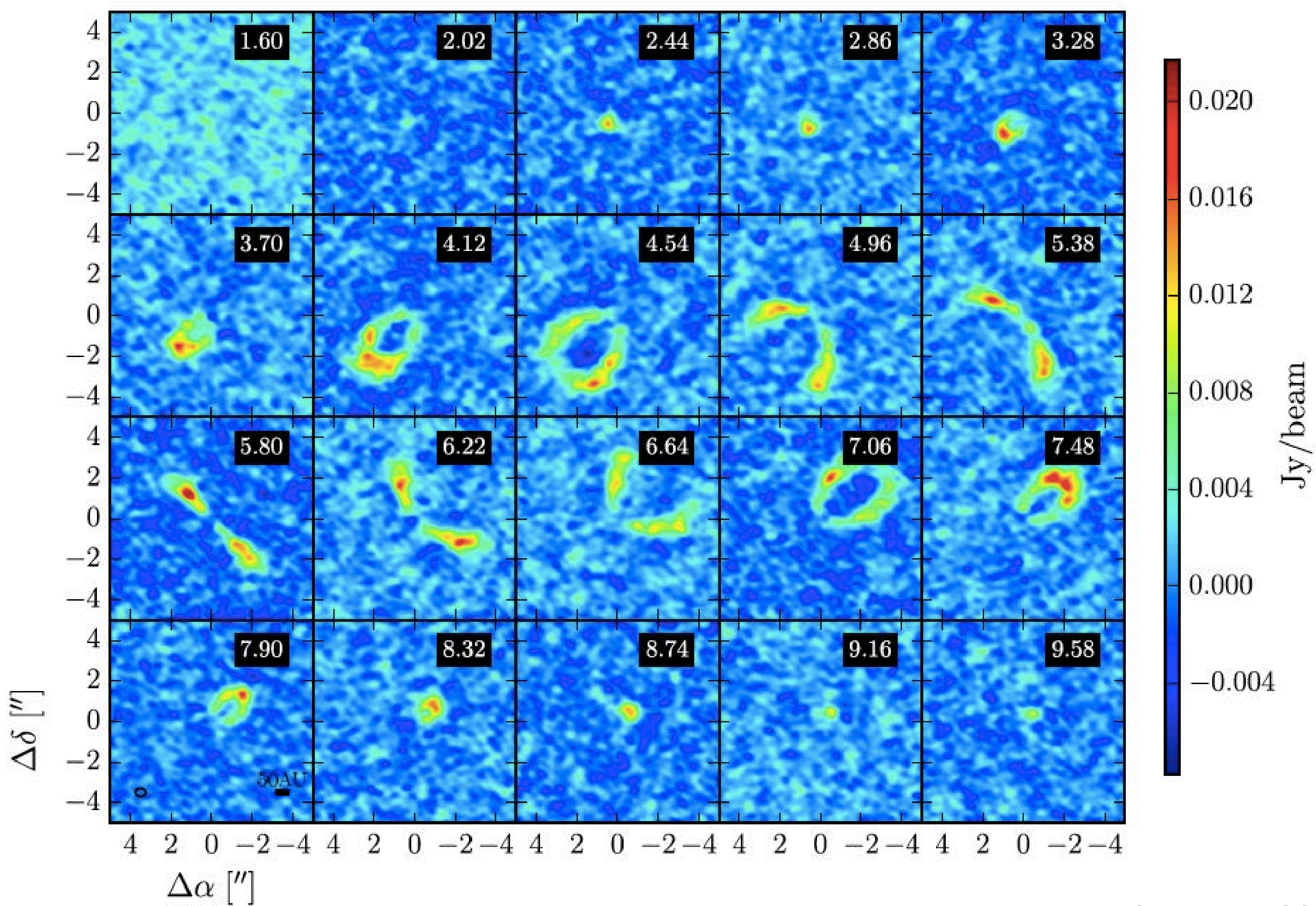


[Loomis + 2018]

Test case #2 – H_2CO in HD 163296



H_2CO 3_{03}-2_{02}



Carney + 2017

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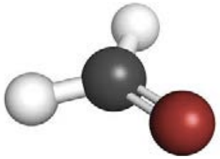
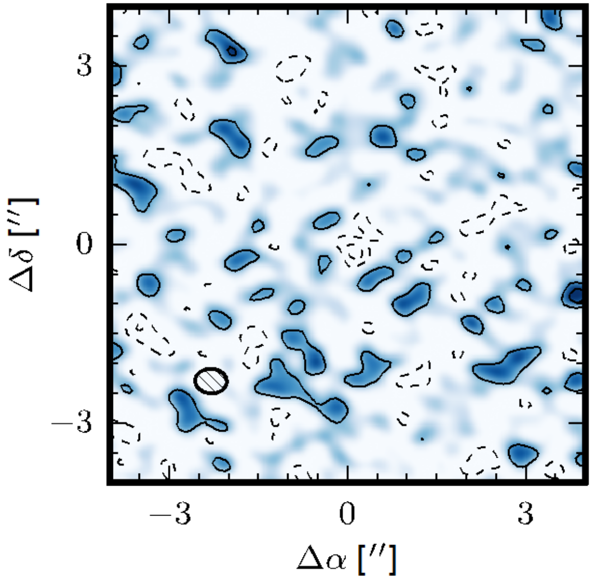
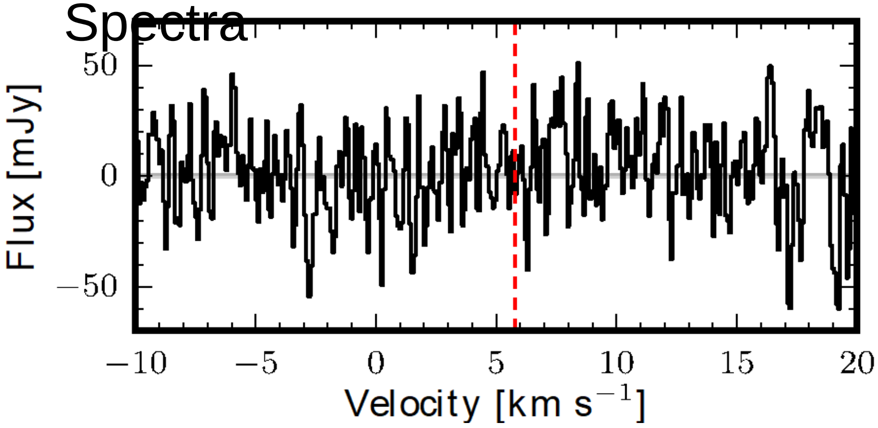


3_{22}-2_{21} + 3_{21}-2_{20} Moment-0



H_2CO 3_{22}-2_{21} + 3_{21}-2_{20} Stacked Spectra



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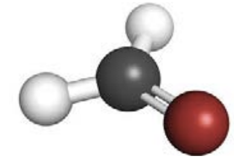

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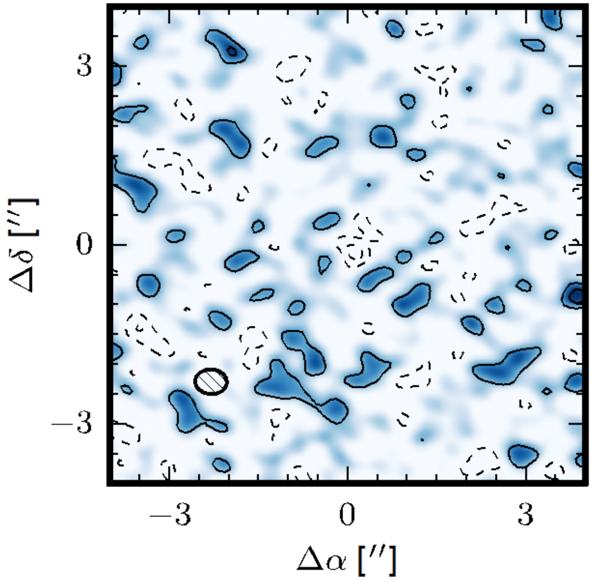
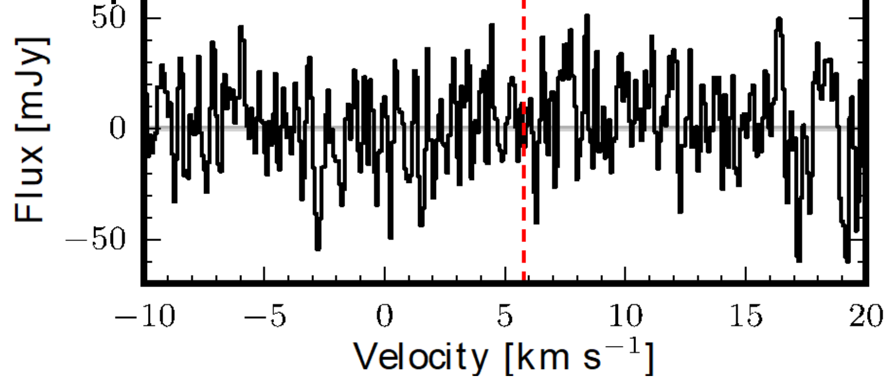


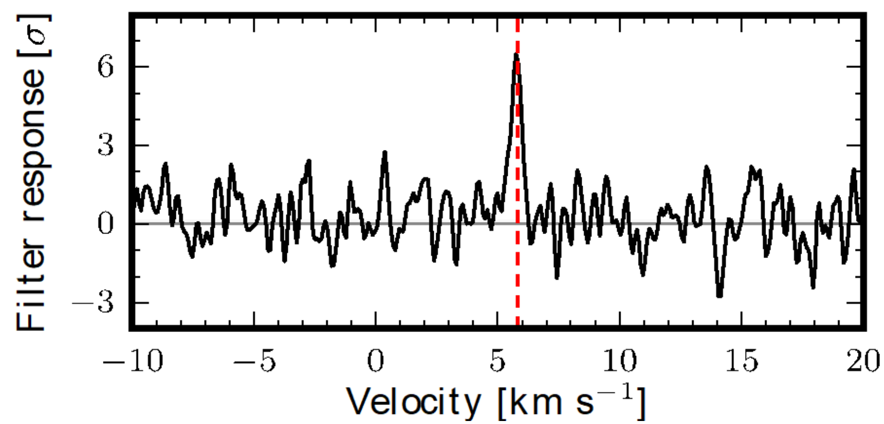
>500% SNR boost!

Carney + 2017

H_2CO 3_{22}-2_{21} + 3_{21}-2_{20} Stacked Spectra



Matched filter



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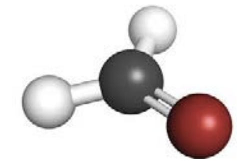
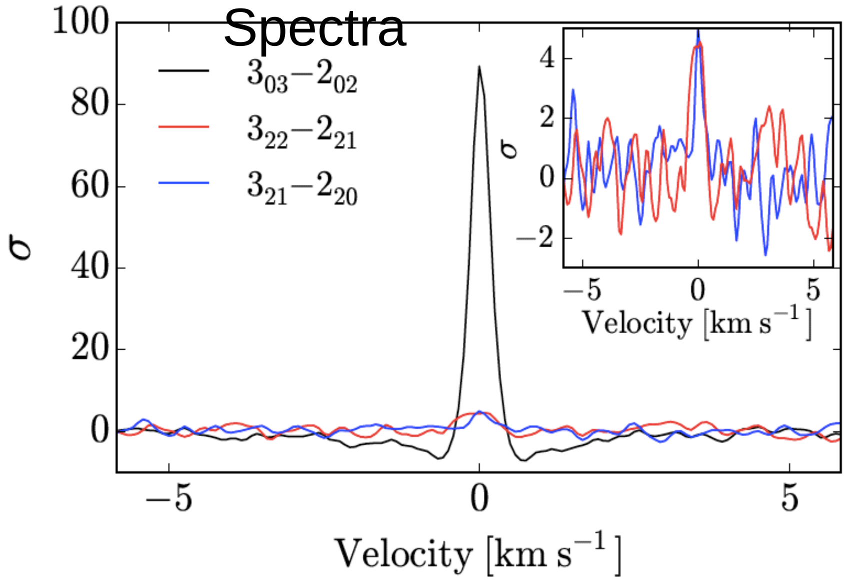
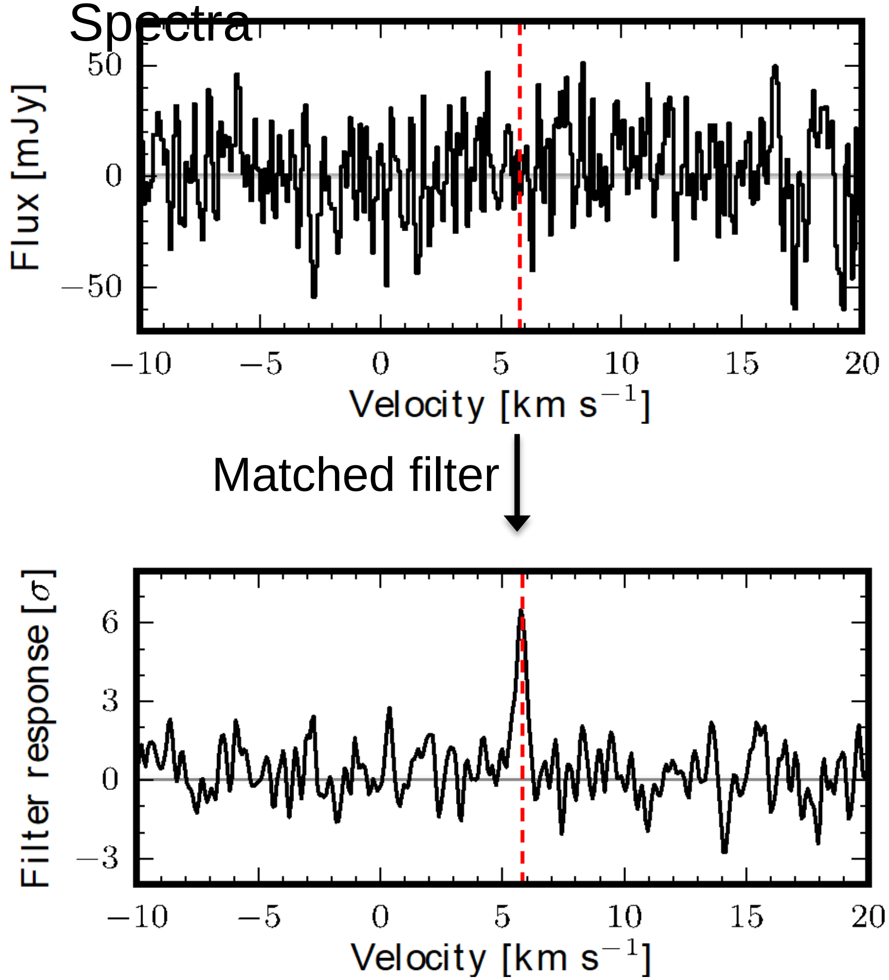


H_2CO Response Spectra



H_2CO 3_{22}-2_{21} + 3_{21}-2_{20} Stacked Spectra



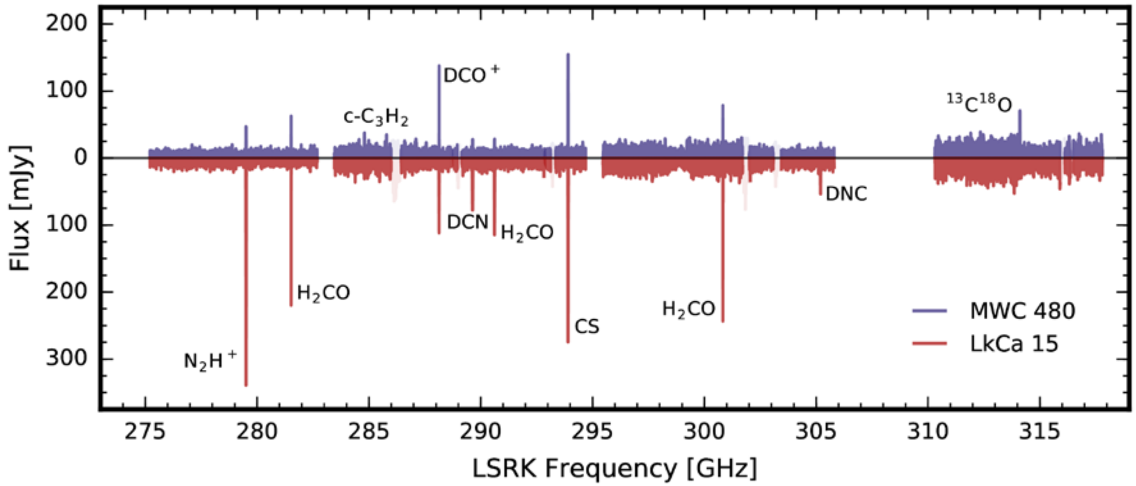
>500% SNR boost!

Carney + 2017

ALMA Band 7 Disk Spectral Survey

Aperture-extraction

- ~40-50 hrs to image, clean, and extract spectra
- 10 detected species



[Loomis + in prep]

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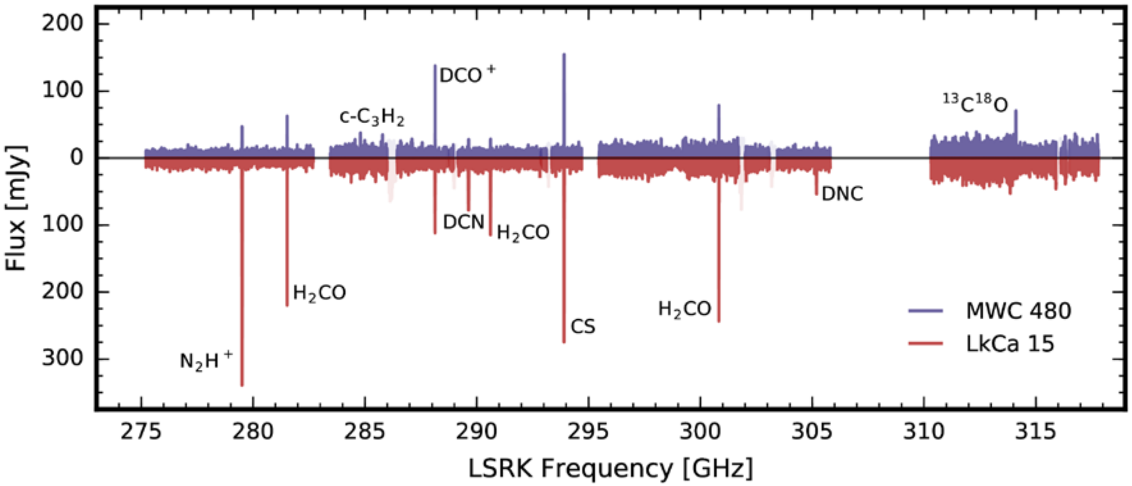
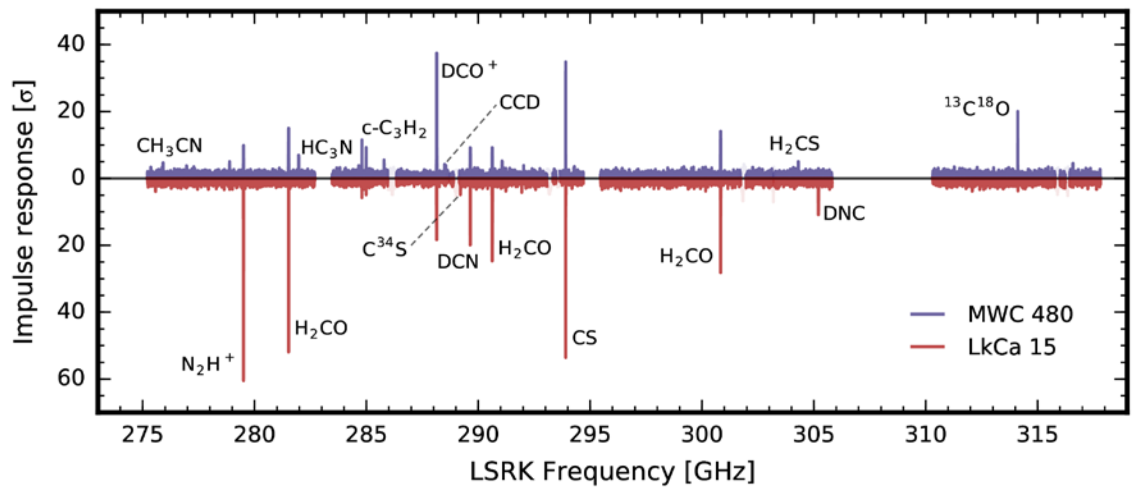


Matched filtering

- ~30 min to filter entire survey
- SNR 2-3x higher
- 16 detected species
- 5 new disk species: ^{13}CS, $C^{34}S$, DNC, CCD, H_2CS



[Loomis + in prep]

Summary

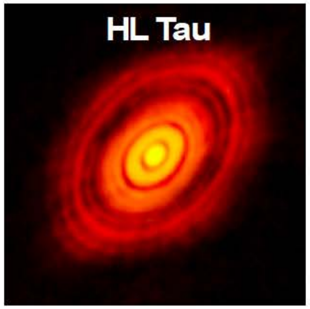
- Matched filtering can boost SNR of weak spectral lines by >500%
- Useful for fast *uv* plane analysis of large bandwidth datasets
  - No prior imaging necessary!
- Not limited to disks – method is general
  - Ideal for resolved sources with PV structure

Outline

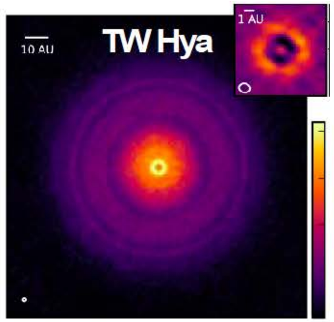
- Matched filtering for improved weak spectral line detection

- Generalized Briggs weighting for improved image fidelity

Motivation

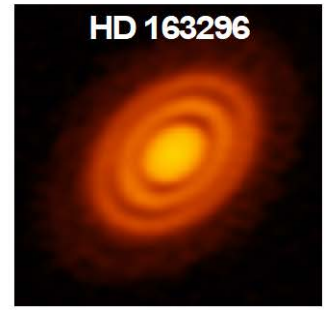


ALMA + (2015)

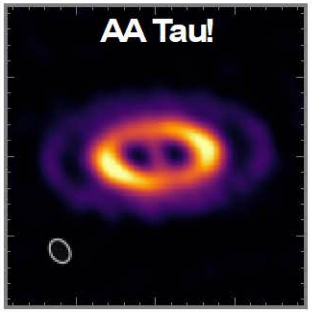


Andrews + (2016)

Perez + (2016)

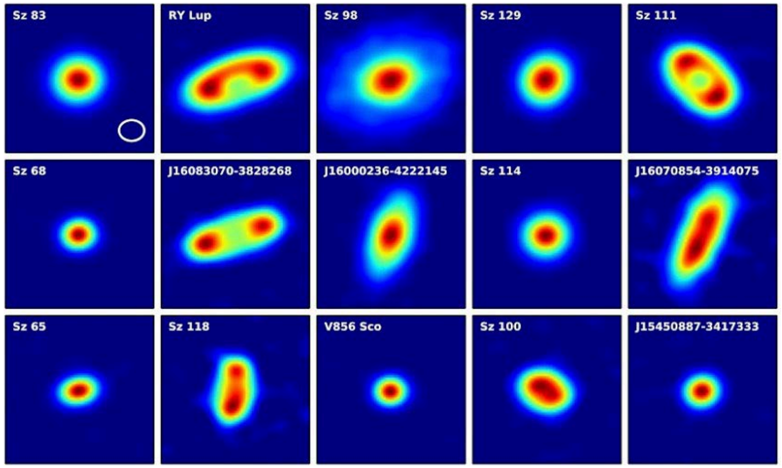


Isella + (2016)



Loomis + (2017)

- Many resolved sources now showing sub-structure
- SNR limitations may prevent imaging this sub-structure in weaker sources

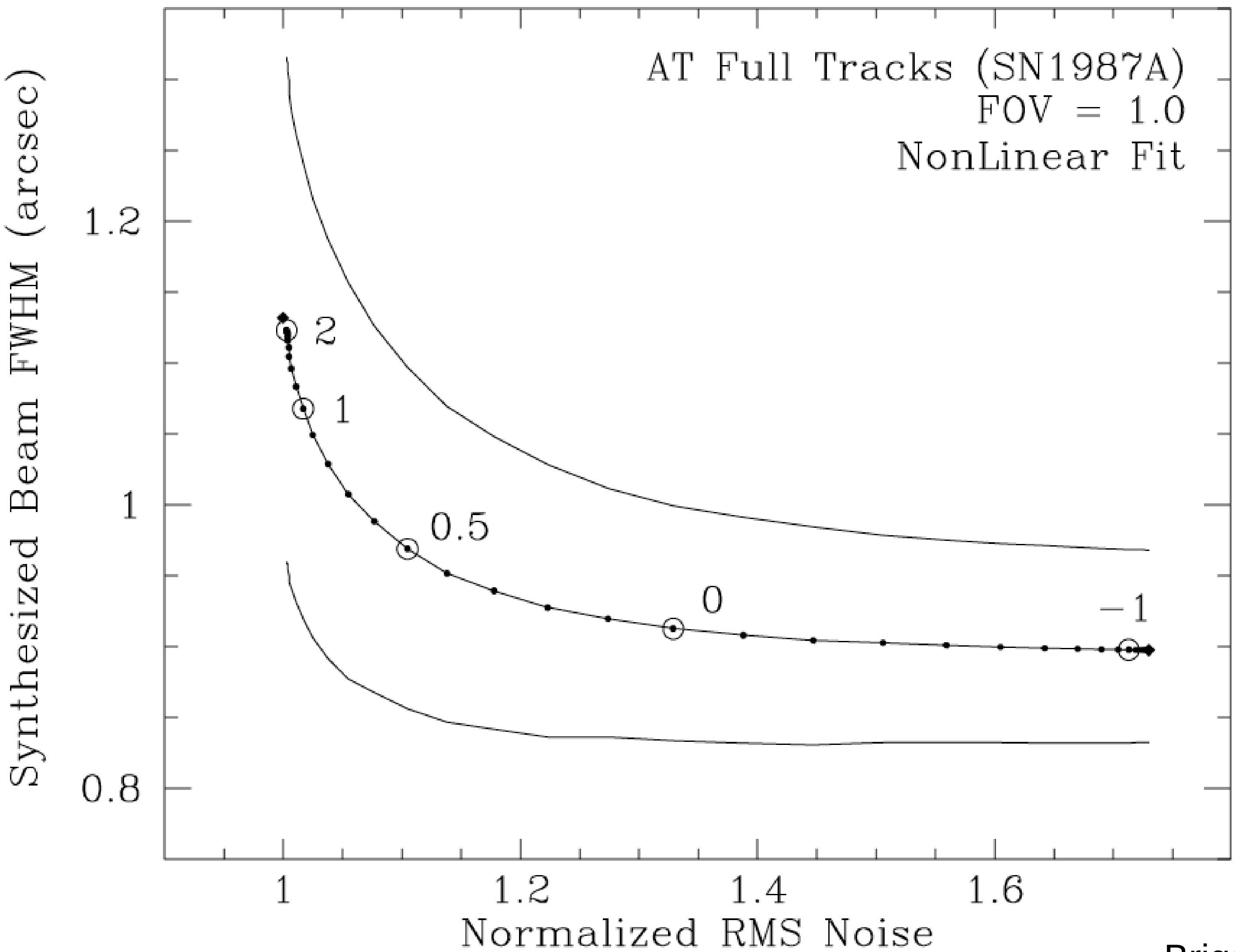


Ansdell + 2016

Types of weighting

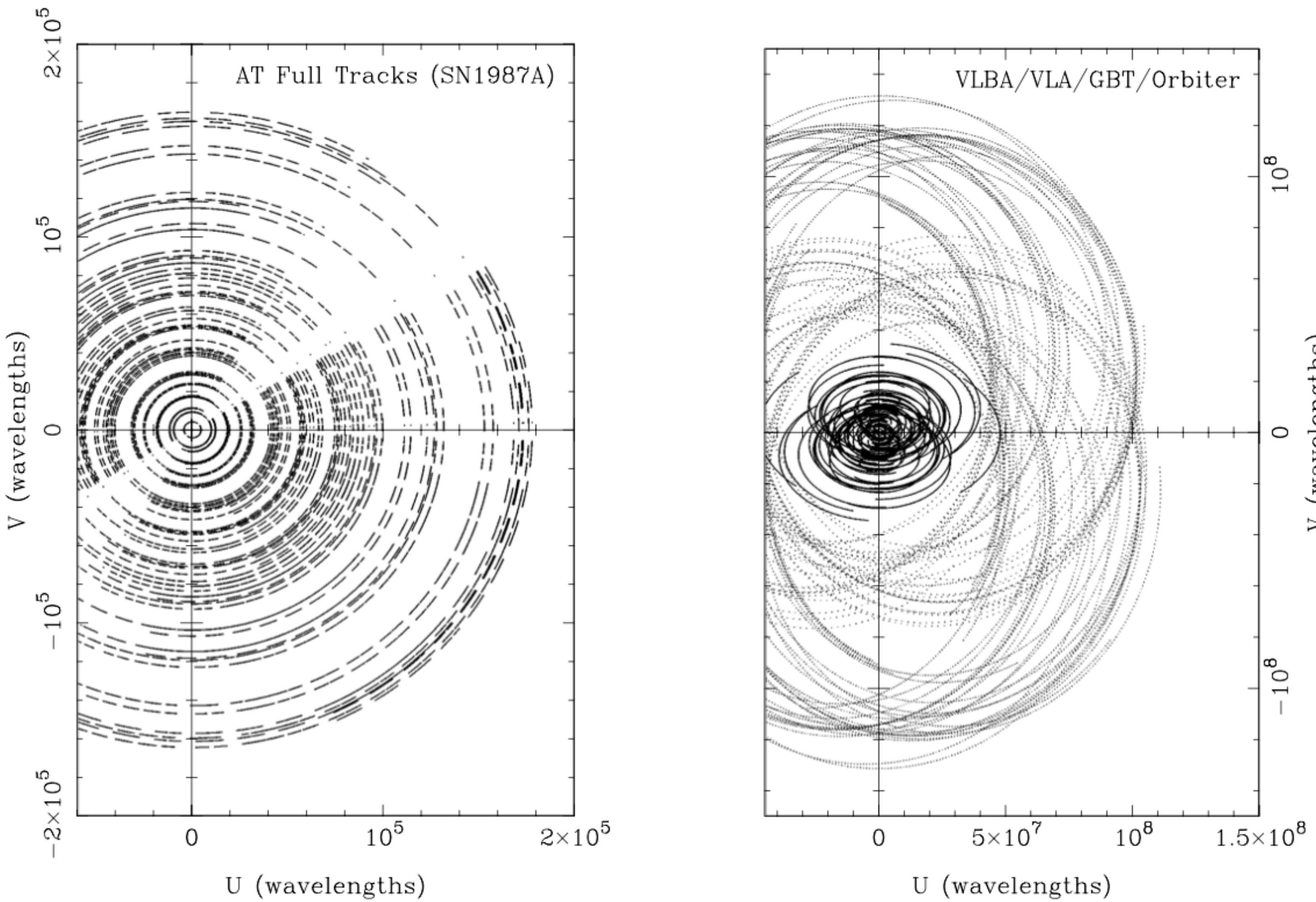
- Natural – lowest rms in dirty image
- Uniform – lowest sidelobes, higher resolution
- Robust – tunable between natural and uniform
  - Retains benefits of both
- Tapering – generally Gaussian taper
  - Sacrifices resolution for increased sensitivity to extended structure

Robust weighting



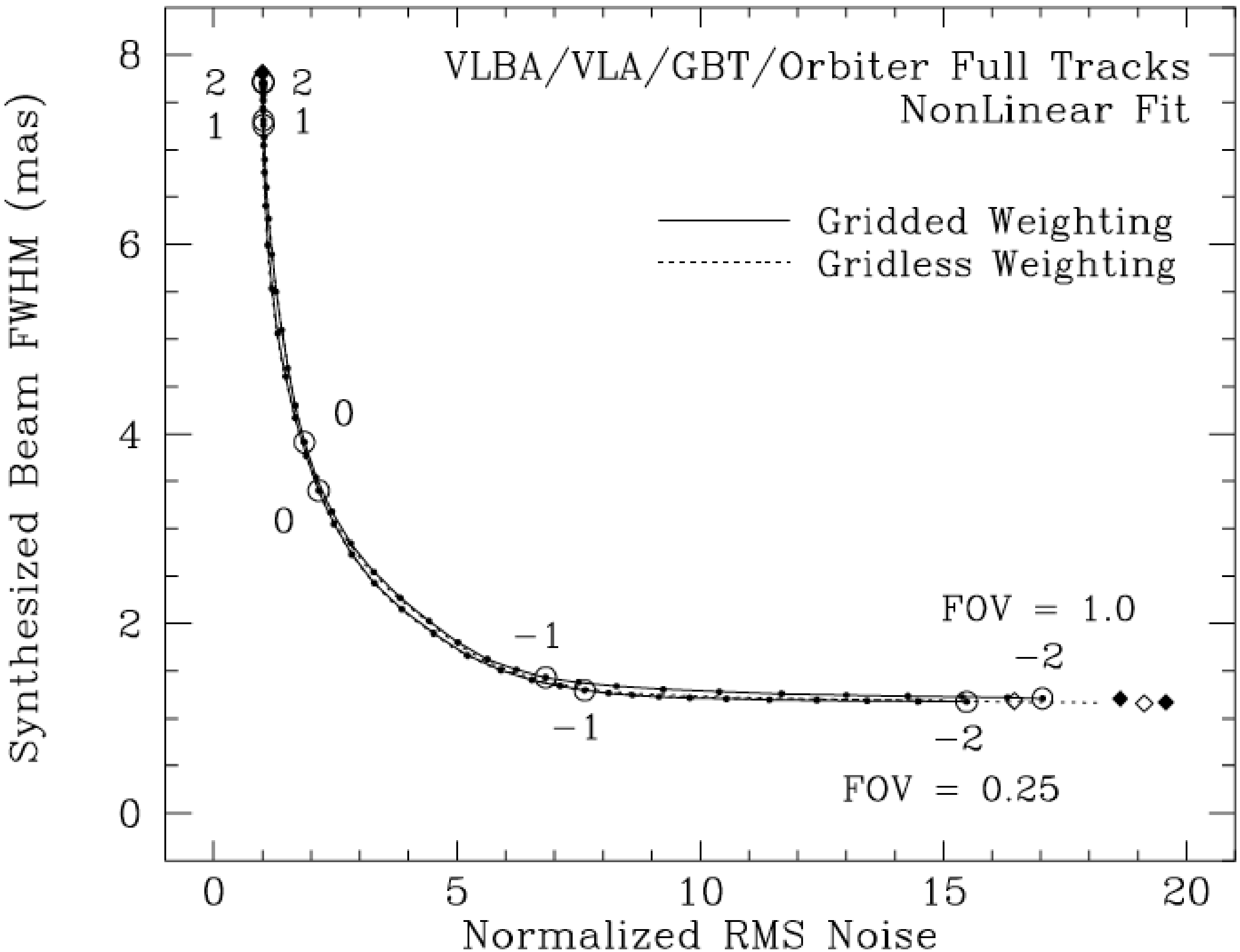
Briggs 1995

UV coverage and array balance



Briggs 1995

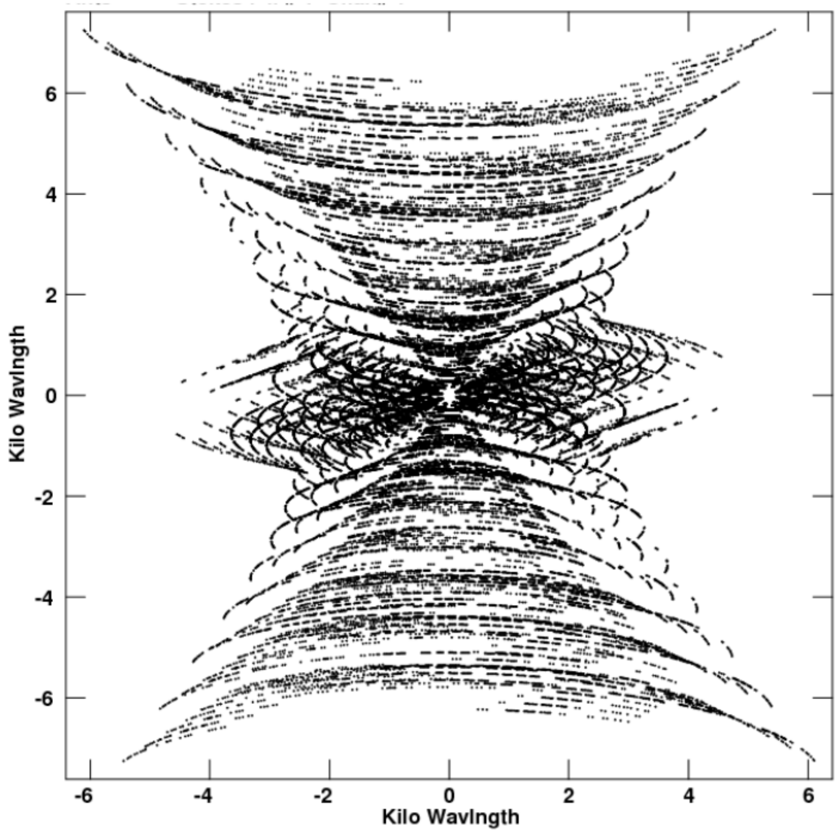
Robust weighting



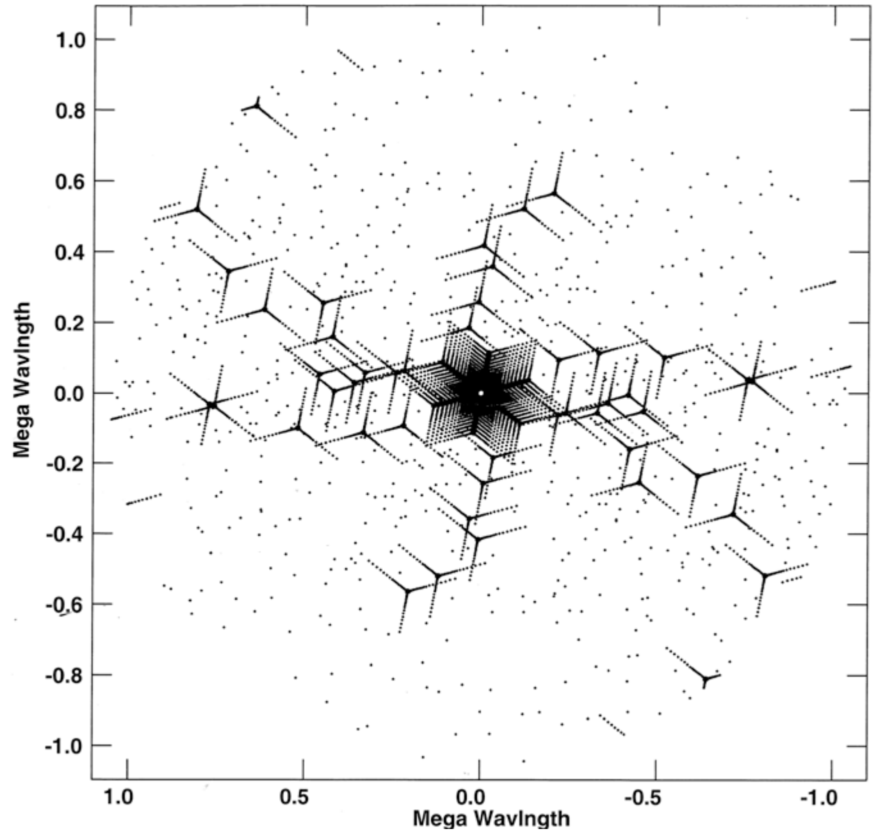
Briggs 1995

UV coverage and array balance

ALMA



ngVLA



Eric Murphy's bulletin board

Why does robust weighting work?

Definitions

Weights: $$W(u,v) = \sum_{k=1}^{2L} T_k D_k w_k \delta(u - u_k, v - v_k)$$

Briggs 1995

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SNR weights = $1/\sigma_k^2$

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Dirty image: $$\begin{aligned} I^D(\ell, m) &= C\,\mathfrak{F}[V^W] = C\,\mathfrak{F}[WV] \\ &= C \sum_{k=1}^{2L} T_k D_k w_k V_k e^{2\pi i (u_k \ell + v_k m)} \end{aligned}$$

Briggs 1995

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Variance of the dirty image: $$(\Delta I^D)^2 = \langle (I^D)^2 \rangle - \langle I^D \rangle^2$$

RMS thermal noise: $$\Delta I^D \propto \sqrt{\sum_{k=1}^{L} T_k^2 D_k^2 w_k}$$

Briggs 1995

Natural weighting

- Minimize $(\Delta I^D)^2$ using Lagrangian multiplier:

$$\frac{\partial}{\partial D_k}\left[\left(\frac{\Delta I^D}{2C}\right)^2 + \lambda\left(\sum T_k D_k w_k - W\right)\right] = 0$$

Briggs 1995

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$$T_k D_k w_k \propto 1/\sigma_k^2$$

- Thermal noise is lowest if T_k and $D_k = 1$, as $w_k = 1/\sigma_k^2$

Briggs 1995

Uniform weighting

- Minimize sidelobes by minimizing beam power $||B||^2$:

$$\frac{\partial}{\partial W_{pq}}\left[N_\ell N_m W^2 \, ||B||^2 + \lambda \left(\sum_{p,q} W_{pq} - W\right)\right] = 0$$

Briggs 1995

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$$D_k=\begin{cases} \mathrm{const}\Big/\sum\limits_{k\in c_{pq}} w_k & \sum\limits_{k\in c_{pq}} w_k>0 \\ 0 & \text{otherwise}\end{cases}$$

Briggs 1995

Robust weighting

- Find a smooth knob between natural and uniform weighting through minimization

$$V_{pq} = S + \varepsilon_{pq}$$

$$\left\langle \|I^D\|^2 \right\rangle = \frac{1}{N_\ell N_m W^2} \sum_{p,q} W_{pq}^2 (S^2 + 2\sigma_{pq}^2)$$

$$W_{pq} \propto \frac{1}{S^2 + 2\sigma_{pq}^2}$$

Briggs 1995

‘Natural’ weighting for a resolved source

- Natural weighting is only ideal for a point source

Briggs 1995

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- Maximize SNR for an arbitrary source: *V*

SNR:
$$\frac{I^D(0,0)}{\Delta I^D} = \frac{\sum_{j=1}^{L} T_j D_j w_j \mathrm{Re}(\mathcal{V}'_j)}{\left(\sum_{k=1}^{L} T_k^2 D_k^2 w_k^2 \sigma_k^2\right)^{1/2}}$$

Briggs 1995

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$T_k D_k w_k \propto 1/\sigma_k^2$ becomes: $T_k D_k w_k \propto \mathrm{Re}(V_k)/\sigma_k^2$

Briggs 1995

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- Gaussian tapering increases SNR of resolved sources *because the taper approximates a matched filter*

$$T_k D_k w_k \propto |V_k|/\sigma_k^2$$

Briggs 1995

Generalized Briggs robust weighting

- Can we find a way to connect matched filter weighting with uniform weighting?

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- Minimize: $f(W) \propto \frac{1}{\mathrm{SNR}} + \alpha \left\| B \right\|^2$

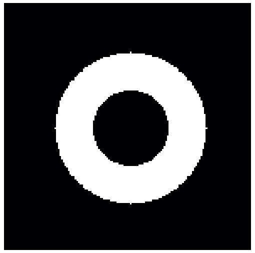
Generalized Briggs robust weighting

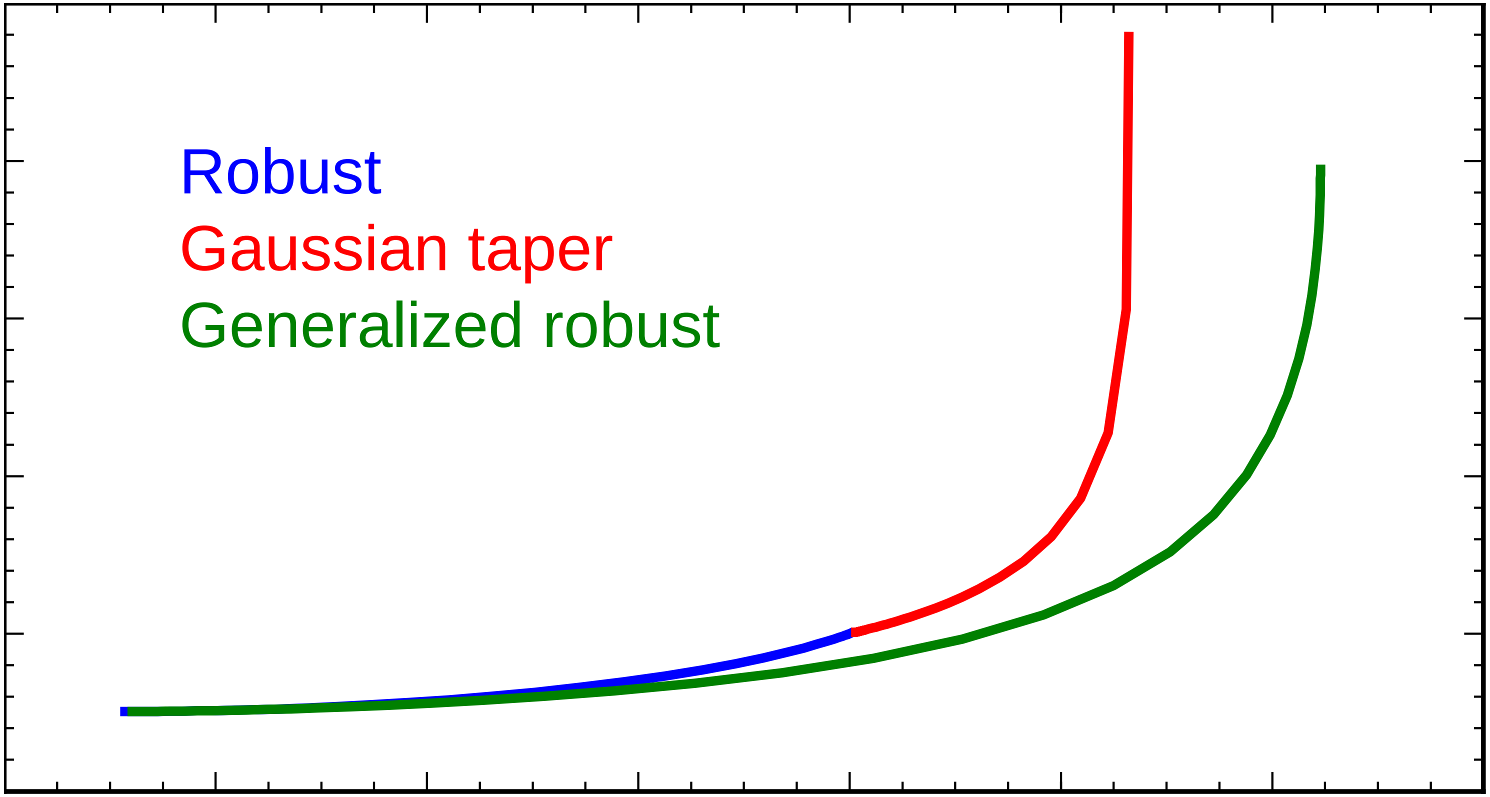
- Can we find a way to connect matched filter weighting with uniform weighting?

- Minimize: $f(W) \propto \frac{1}{\mathrm{SNR}} + \alpha \left\| B \right\|^2$

- Find that: $W_{pq} = \frac{(5 \times 10^{-r})^2 \, \overline{|V_{pq}|} + |V_{pq}|}{(5 \times 10^{-r})^2 \, \overline{\sigma_{pq}^2} + \sigma_{pq}^2}$

Test case - ring





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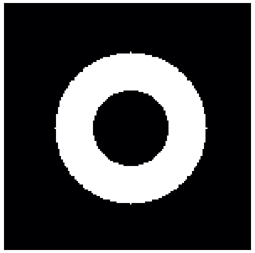
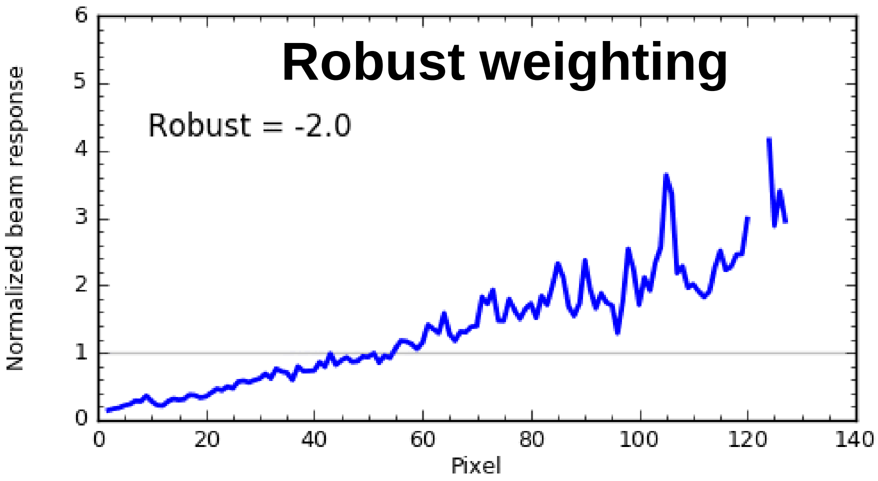
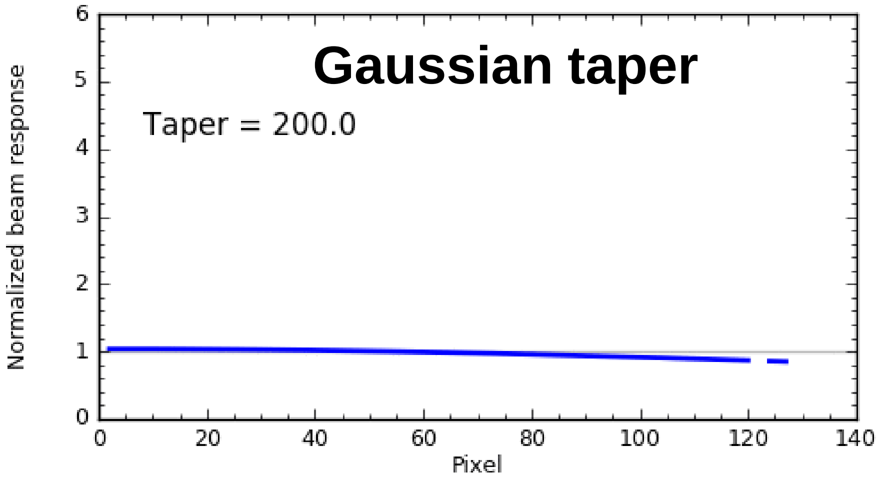
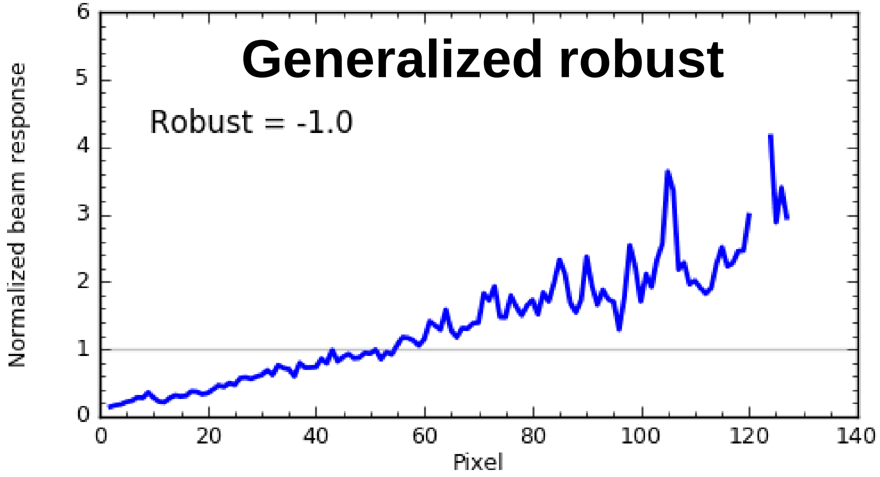
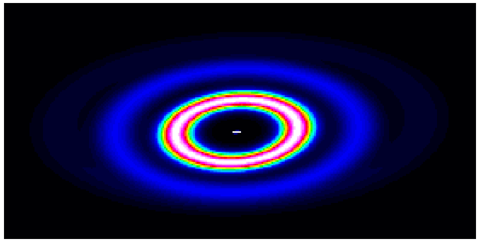


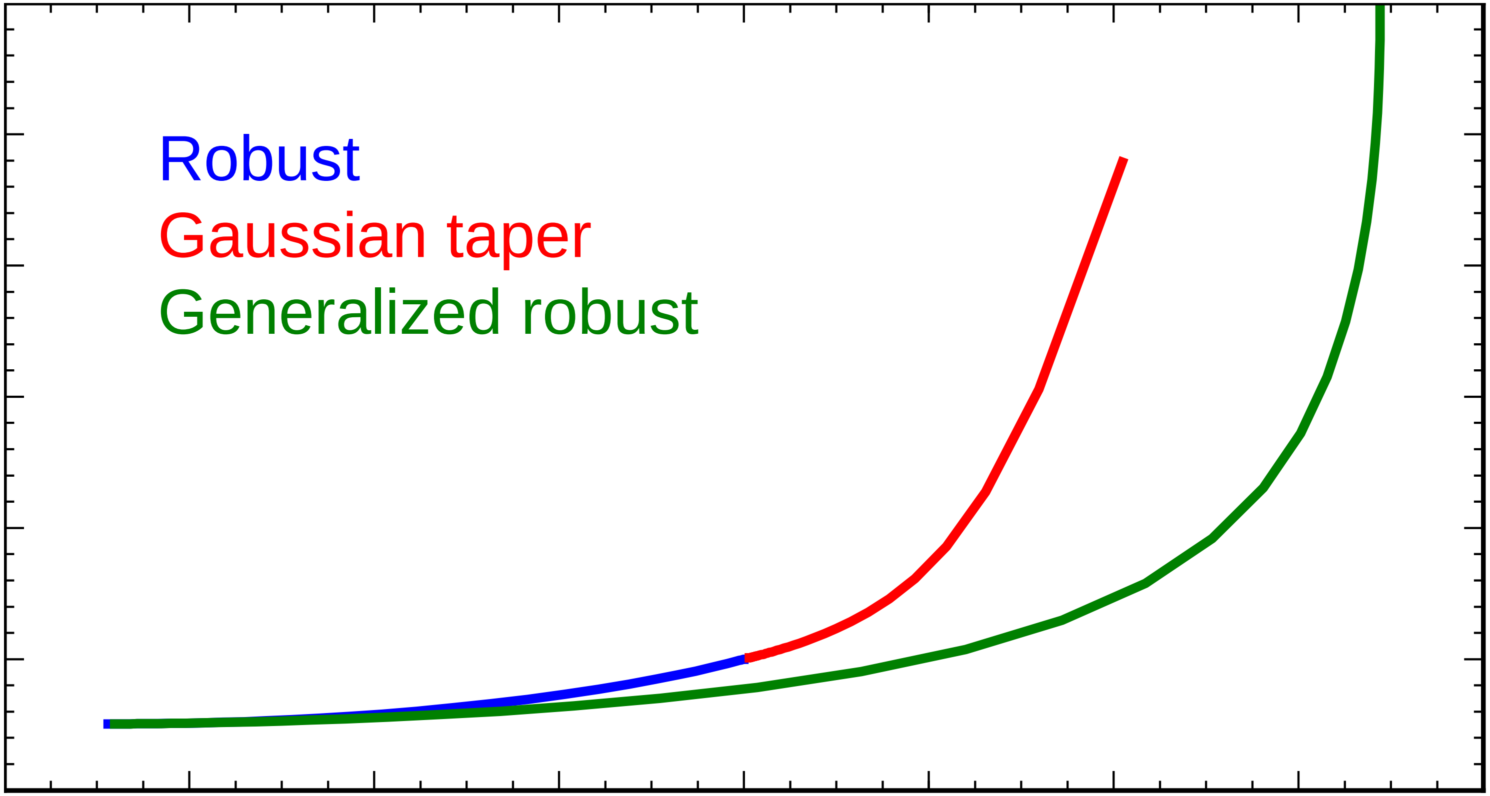






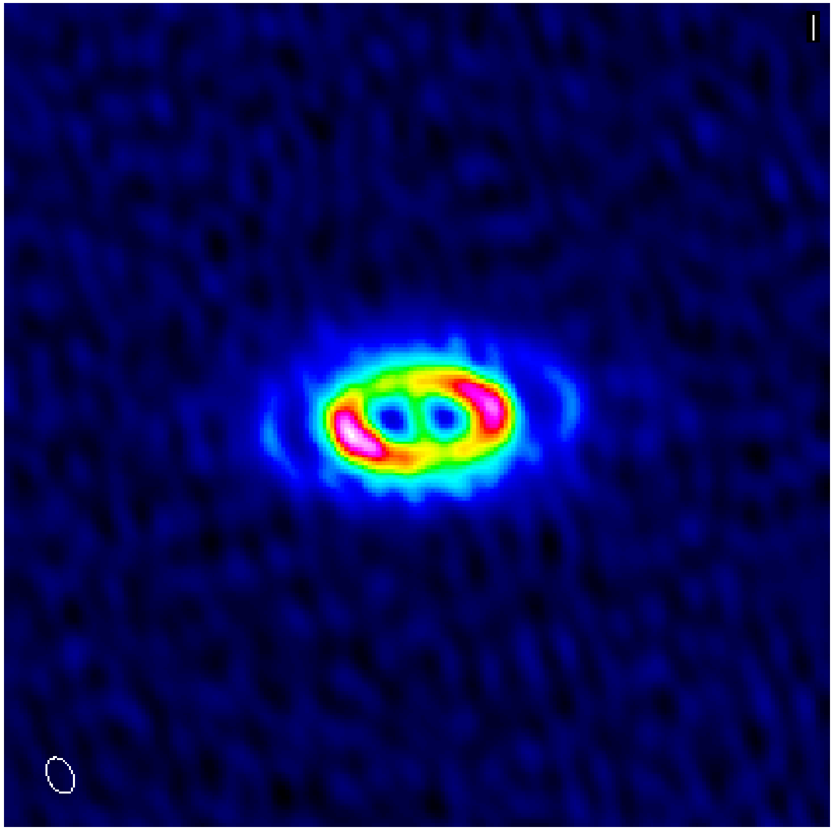
Realistic example – AA Tau





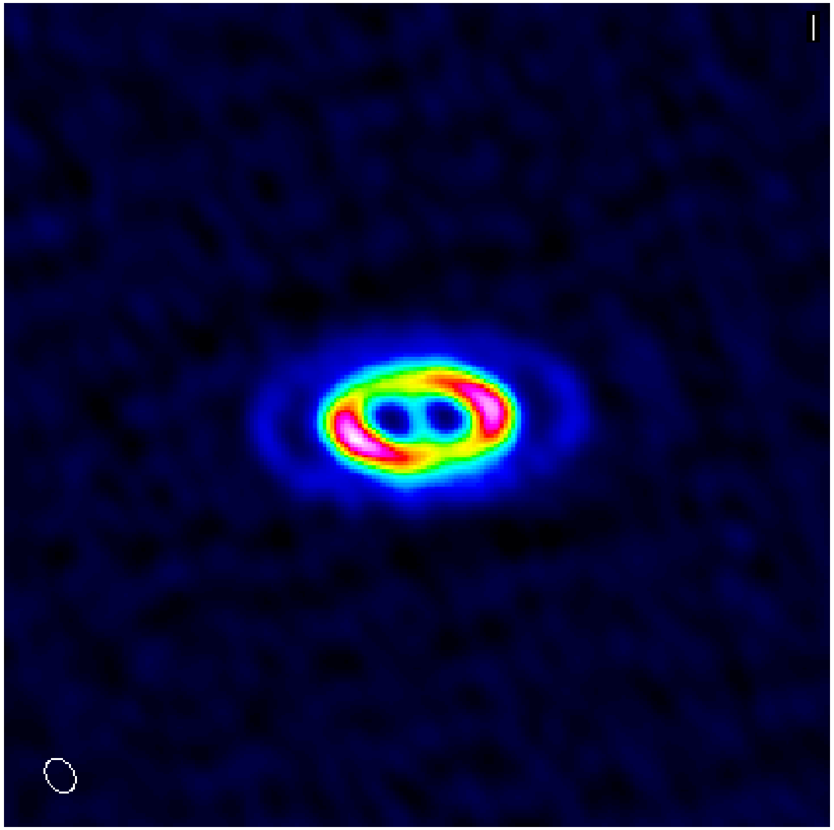
Realistic example – AA Tau

Uniform weighting



SNR = 49

Generalized Briggs



SNR = 87

Loomis + 2017

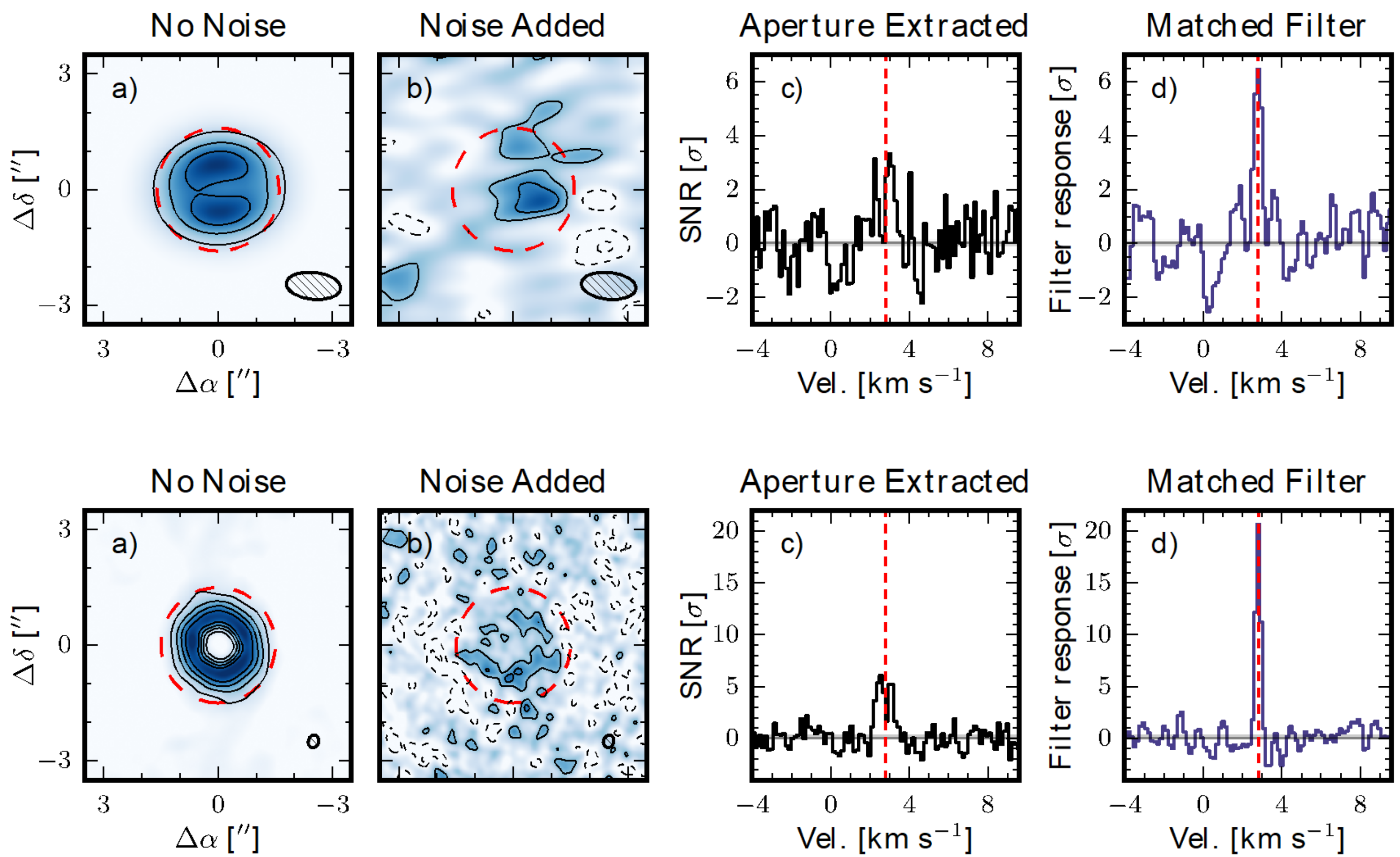
Summary

- Generalized robust weighting outperforms robust weighting for all extended sources, and reduces to robust weighting for point sources
- Especially shines in comparison to tapering, good for cases where you need a little extra SNR, but want to preserve resolution
- Need knowledge of source structure – CLEAN model is an estimate of this!

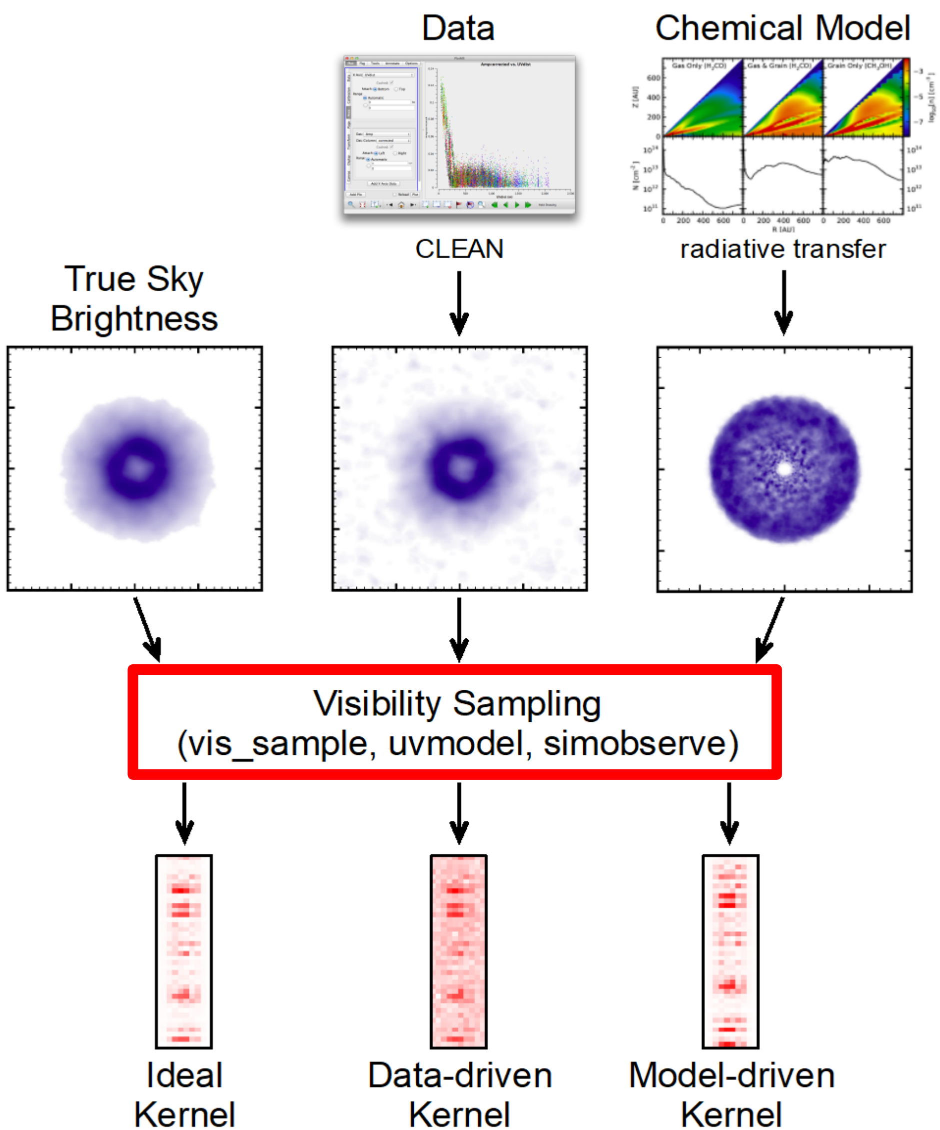
Summary

- Matched filtering allows priors to be leveraged, increasing spectral SNR and image fidelity
- Prior constraints can be derived from the data itself, or from astrophysical/chemical models
- VISIBLE tool available for use! Check it out at: github.com/AstroChem/VISIBLE
- Generalized robust weighting implemented in CASA, stay tuned for release

SNR boost is proportional to resolution



Kernel generation



vis_sample

- Python implementation of uvmodelhttps://github.com/AstroChem/vis_sample
- ~25% faster than uvmodel
- Accepts FITS cubes (i.e. from LIME) and RADMC-3D output files
- Interfaces with CASA measurement sets (input and output)